Quadratic Equations

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QUADRATIC EQUATION

- is an equation that contain a second-degree term and no term of a higher degree in the form,

$$ax^2 + bx + c = 0$$

where a, b, and c, are real numbers; and a ≠ 0.

Examples

1. $x^2 - x + 4 = 0$
2. $2x^2 + 5 = 0$
3. $x + 3x^2 - 1 = 0$
4. $-4x^2 + 6 = 0$

Finding the Roots of a Quadratic Equation

- a number is a ***root*** of a quadratic equation if, when the number is substituted for the variable, the equation becomes a true statement.

METHODS OF SOLVING QUADRATIC EQUATIONS

- Extracting the Square Roots
- Factoring Method
- Completing the Square
- Quadratic Formula

Solution by Extracting the Square Roots

- To solve a quadratic equation of the form $x^2 = k$, where k = 0,
  - Solve the equation for the square of the unknown number.
  - Find the square roots of both members of the equation.

Theorem:

Let $k \geq 0$. If $x^2 = k$, then

$$x = \sqrt{k} \text{ or } x = -\sqrt{k}.$$

Note:

$x = \sqrt{k}$ or $-\sqrt{k}$ can be written as $x = \pm\sqrt{k}$.

Example

1. $x^2 - 121 = 0$

Solution:

Applying the addition property of equality, we have

$$x^2 = 121$$

$$\sqrt{x^2} = \sqrt{121}$$

$$x = \pm 11$$

Example

2. $\mathbf{3x^2 - 27 = 0}$

Applying the addition and multiplication property of equality, divide both side by 3, we have

$$3x^2 = 27$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

Solution of a Quadratic Equation by Factoring

Theorem:

Zero-Product Property of Real Numbers

If a and b are real numbers, then

$$ab = 0$$

if, and only if, $a = 0$ or $b = 0$.

Solution of a Quadratic Equation by Factoring

Consider the equation 7xy = 0. This means that either x = 0, y = 0 or both x and y are equal to 0.

NOTE: If $ax^2 + bx + c$ is factorable, then $ax^2 + bx + c = 0$ can be solved by ***factoring.***

STEPS

- **To solve a quadratic equation by factoring:**
  1. Transform the given equation to the form $ax^2 + bx + c = 0$.
  2. Factor the left member of the equation.
  3. Equate each factor to zero.
  4. Solve each resulting equation.
  5. Check in the original equation.

Example

1. Solve by factoring: $x^2 - 6x = -8$.

Solution:

$x^2 - 6x + 8 = 0$ Transform to standard form.

$(x - 4)(x - 2) = 0$ Factor the left member.

$x - 4 = 0$ $x - 2 = 0$ Use the Z-P Property.

$x = 4$ $x = 2$

Check: If $x = 4$ If $x = 2$

$16 - 24 + 8 = 0$ $4 - 12 + 8 = 0$

$0 = 0$ $0 = 0$

Solution By Completing the Square

- Quadratic equations that cannot be easily solved by factoring can be solved by using completing the square method.

Solution By Completing the Square

- A ***perfect square trinomial*** is a trinomial that can be expressed as the square of a binomial.

Examples

The following are examples of perfect square trinomials:

$$x^2 + 4x + 4 = (x + 2)^2$$

$$x^2 - 6x + 9 = (x - 3)^2$$

STEPS

To solve a quadratic equation by completing the square:

1. Transform the equation to the form

$$ax^2 + bx = c, \text{ where } a = 1.$$

2. Add to each member of the equation the square of half the coefficient of x.

STEPS

3. Find the square root of each member of the equation, writing the double sign “±” before the square root of the right member.
4. Solve the resulting linear equation.
5. Check in the original equation.

Example

1. Solve for x : $x^2 - 2x - 2 = 0.$

Solution:

$$x^2 - 2x = 2$$

Separate the constant from terms containing x.

$$x^2 - 2x + 1 = 2 + 1$$

Add $\left(\frac{-2}{2}\right)^2$ or 1 to both sides of the equation to form a perfect square trinomial.

Example

Cont…

$(x-1)^2 = 3$

Rewrite the left member as a square of a binomial.

$x - 1 = \pm\sqrt{3}$

Extract the square root of both sides and prefix ± sign to check to the result on the right member.

Example

Cont…

Solve the resulting linear equations and check:

$$x - 1 = \pm\sqrt{3} \qquad x - 1 = -\sqrt{3}$$

$$x = 1 + \sqrt{3} \qquad x = 1 - \sqrt{3}$$

Example

2. Solve for x: $2x^2 - 3x - 2 = 0$

Solution:

$$\frac{2x^2}{2} - \frac{3x}{2} - \frac{2}{2} = 0$$

$$x^2 - \frac{3x}{2} = 1$$

$$x^2 - \frac{3x}{2} + \frac{9}{16} = 1 + \frac{9}{16}$$

Add $\left[\frac{\left(-\frac{3}{2}\right)}{2}\right]^2$ or $\frac{9}{16}$ to both sides of the equation to form a perfect square trinomial.

Example

Cont…

$$x-\frac{3}{4}=\pm\sqrt{\frac{25}{16}}$$

Extract the square root of both sides.

$$x=\frac{3}{4}\pm\frac{5}{4}$$

Solve each resulting equation.

Solution set: $\{2,-\frac{1}{2}\}$.

Solution by Using the Quadratic Formula

The quadratic formula states that if $ax^2 + bx + c = 0$, with a $\neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

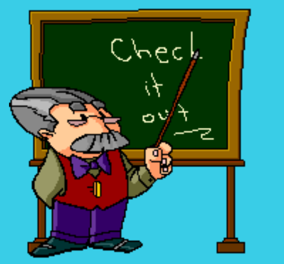
1. Solve the equation $x^2 - 3x + 1 = 0$.

Solution:

a = 1, b = -3, and c = 1

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

Example



Cont…

$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

The roots of the equation are $\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\}$.

WORD PROBLEM

Example:

1. If the square of a number is added to 3 times to the number, the sum is 108. Find the number.

WORD PROBLEM

Solution:

Let x be the number.

$x^2 + 3x = 108$

$x^2 + 3x - 108 = 0$

(x + 12) (x - 9) = 0

x + 12 = 0 or x - 9 = 0

x = -12 x = 9

The number is 9 or -12.

WORD PROBLEM

2. The speed at which water travels in a pipe can be measured by directing the flow through an elbow and measuring the height it spurts out on the top. If the elbow's height is 10 cm, the equation relating to the height of the water above the elbow (in cm) and its velocity v (in cm/sec) is given by $v^2 = 1960\,(h + 10)$. Find v if h = 2 cm.

WORD PROBLEM

- ***Solution:***

Substitute the value of h in the formula:

$$v^2 = 1960\ (2 + 10)$$

$$= 1960\ (12)$$

$$= 23,\ 520$$

$$V = \sqrt{23\ 520} = 153.36 \text{ cm/sec}$$

WORD PROBLEM

Direction: Solve the following problems.

1. Martha had a square patio. After expanding the length by 1.5 m and the width by 0.5 m, the area become 20 m^2. What was the original area of the patio?

WORD PROBLEM

Direction: Solve the following problems.

1. A square piece of cardboard is to be used to form a box without a top by cutting off squares, 5 cm on a side from each corner and then folding up the sides. If the volume of the box must be 320 cm^3, what must be the length of a side of the cardboard.

Thank you