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MATH 11N: MATHEMATICS IN THE MODERN WORLD

Module 3

QUADRATIC EQUATIONS

(3.4. Solutions of Quadratic Equation by Completing the Square)

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Solving Quadratic Equation using Completing the Square Method

Hi, today you will learn solving quadratic equation using completing the square. Before we start, let us consider first the following learning objectives.

Learning Objective

After studying this lesson, you are expected to:

- solve quadratic equation using completing the square method.

Let's Go Online!

I want you to visit the following links and study the materials.

1. https://www.mathplanet.com/education/algebra-1/quadratic-equations/completing-the-square. This link provides a discussion of the concept and with video presentation on solving quadratic equation using completing the square method. Exercises are also provided for mastery of the skill.

2. https://www.mesacc.edu/~scotz47781/mat120/notes/quadratics/comp_squar practice_problems.html. This link provides practice problems which will help you strengthen your skill in solving quadratic equation by completing the square. Try solving each item. A complete solution was also provided for reference.

3. https://www.cliffsnotes.com/study-guides/algebra/algebra-ii/quadratics-in-one-variable/quiz-solving-quadratics-by-completing-the-square. This link provides practice exercises in solving quadratic equations using completing the square. Take the exercises and write your answer in a short bonder paper.

4. https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:completing-square-quadratics/a/solving-quadratic-equations-by-completing-the-square. This links provides a comprehensive discussion of solving quadratic equation by completing the square. Read and study the material.

After watching the videos and studying the materials online, use your *GoConqr* account and answer the pre-assessment activity.

How do you find our lesson online? Is there anything you'd like to clarify about the presentation? Do you have listed all your concerns and clarifications? Let us discuss it now.

DISCUSSIONS AND ACTIVITIES (Face-to-Face)

Completing the square is a method, usually of solving quadratic equations, by which a quadratic expression is written as the sum or difference of perfect square and a constant by addition and subtraction of appropriate constant terms. Further, when the quadratic equation is difficult to factor out, this method is applicable. In fact, any quadratic equation can be solve using completing the square.

Perfect square is the product of number or polynomial multiplied by itself. For instance, 529 is a perfect square number since $23x23$ =529 or $23^2 = 529$. Likewise, $x^2 - 64 = 0$ and $x^2 - 16x + 64 = 0$ are perfect square binomial and trinomial, respectively.

In this lesson, you will learn how to solve quadratic equation using completing the square method.

Pre-requisites

The prior knowledge and skills for this lesson are the following:

- ✓ The concept of squaring a number and definition of perfect square;
- ✓ The concept/rule of radicals can be applied to a particular given; and
- ✓ The concept of sum and difference of perfect squares.

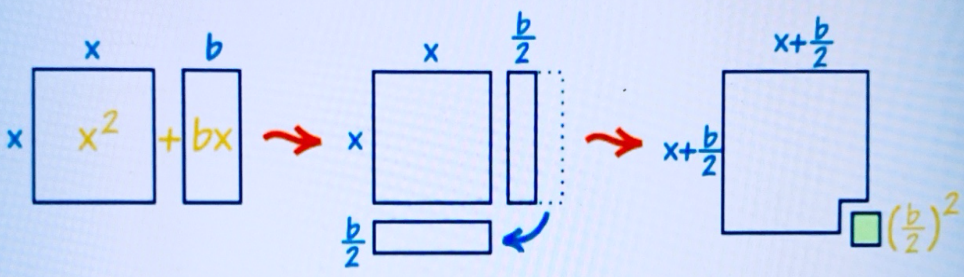
Solving quadratic equation using the method of completing the square is where we take a quadratic equation $ax^2 + bx + c = 0$ and turn it into $a(x + d)^2 + e = 0$ where d is equal to b divided by $2a$, and e is equal to c subtracted by the square of b divided by four times a.

Before using the method completing the square, you need to arrange and write the quadratic equation into its standard form. You have to separate the variable terms from the constant term. Make sure that the coefficient of the first term in standard form is 1. If not, then you need to make it 1 by dividing the value of a, $a > 1$ to each term both sides of the equation. Finally, apply mathematical rules and properties if necessary. Always leave your answer in its simplest form.

Completing the Square

Say we have a simple expression like $x^2 + bx$. Having x twice in the same expression can make life hard. What can we do?

Well, with a little inspiration from Geometry we can convert it, like this:



As you can see $x^2 + bx$ can be rearranged ***nearly*** into a square ...

... and we can **complete the square** with $(b/2)^2$

In Algebra it looks like this:

$$x^2 + bx \quad + (b/2)^2 \quad = \quad (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square.

And $(x+b/2)^2$ has x only **once**, which is easier to use.

In order to facilitate solving, the following are the procedure to determine the roots of the quadratic equation using completing the square.

STEPS in Solving Quadratic Equation using Completing the Square:

Step 1: Divide all terms by a (the coefficient of the first term x^2)

Step 2: Move the number term $\left(\frac{c}{a}\right)$ to the right side of the equation. If there is no numerical coefficient of the first term other than 1, proceed to getting half the middle term and square it.

Step 3: Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation.

Examples

Directions: Solve each of the following quadratic equations by completing the square method.

1) $x^2 + 2x - 15 = 0$

2) $x^2 + 5x = -6$

3) $4x^2 - 2x - 5 = 0$

4) $x^2 + 6x - 7 = 0$

5) What is the two-digit number such that the product of its digit is 12, when 9 is added to the number, the digits were exactly interchange their places?

Solutions:

1) $x^2 + 2x - 15 = 0$ ⇨ roots: $x = \{3, -5\}$

Step 1: Retain $x^2 + 2x - 15 = 0$ since the coefficient of the first term is 1.

Step 2: Instead, move -15 to the right side of the equation, so, $x^2 + 2x = 15$.

Step 3. $x^2 + 2x + \left(\frac{2}{2}\right)^2 = 15 + \left(\frac{2}{2}\right)^2$

$$x^2 + 2x + 1 = 15 + 1$$
$$(x+1)(x+1) = 16$$
$$(x+1)^2 = 4^2$$
$$\sqrt{(x+1)^2} = \pm\sqrt{4^2}$$

$(x+1) = \pm 4$ which means that $x + 1 + (-1) = +4 + (-1) = 3$ and $x + 1 + (-1) = -4 + (-1) - 5$.

Therefore, the solution set of $x^2 + 2x - 15 = 0$ is $\{-5, +3\}$.

2) $x^2 + 5x = -6$ ⇨ $x = \{-2, -3\}$

Step 1: Done, since the coefficient of the first term is 1.

Step 2: Done, since the constant or the third term is in the right side of the equation already.

Step 3. $x^2 + 5x + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2$

$$x^2 + 5x + \frac{25}{4} = -6 + \frac{25}{4}$$
$$\left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right) = -\frac{24}{4} + \frac{25}{4}$$
$$(x + \frac{5}{2})^2 = \frac{1}{4}$$
$$\sqrt{(x + \frac{5}{2})^2} = \pm\sqrt{\frac{1}{4}}$$

$(x + \frac{5}{2}) = \pm\frac{1}{2}$ $x = \{-2, -3\}$

Therefore, the solution set of $x^2 + 5x = -6$ is $\{-2, -3\}$.

3) $4x^2 - 2x - 5 = 0$ ⇨ roots: $x = \left\{\frac{1+\sqrt{21}}{4}, \frac{1-\sqrt{21}}{4}\right\}$

Step 1: Given $4x^2 - 2x - 5 = 0$, divide each term by the coefficient of the first term which is 4. Hence, $x^2 - \frac{1}{2}x - \frac{5}{4} = 0$.

Step 2: Next, move $-\frac{5}{4}$ to the right side of the equation, so that $x^2 - \frac{1}{2}x = \frac{5}{4}$.

Then add $\left(-\frac{1}{4}\right)^2$ or $\frac{1}{16}$ to both sides of the equation.

Step 3. $x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{5}{4} + \frac{1}{16}$

$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{20}{16} + \frac{1}{16}$

$(x - \frac{1}{4})(x - \frac{1}{4}) = \frac{21}{16}$

$(x - \frac{1}{4})^2 = \frac{21}{16}$

$\sqrt{\left(x - \frac{1}{4}\right)^2} = \pm\sqrt{\frac{21}{16}}$

$(x - \frac{1}{4}) = \pm\frac{\sqrt{21}}{4}$ which is also equivalent to saying $x = \pm\frac{\sqrt{21}}{4} + \frac{1}{4}$. That is, $x = +\frac{\sqrt{21}}{4} + \frac{1}{4}$ and $x = -\frac{\sqrt{21}}{4} + \frac{1}{4}$.

Therefore, the solution set of $4x^2 - 2x - 5 = 0$ is $\left\{\frac{1+\sqrt{21}}{4}, \frac{1-\sqrt{21}}{4}\right\}$.

4) $x^2 + 6x - 7 = 0$ ⇨ roots: $x = \{1, -7\}$

Step 1: Retain $x^2 + 6x - 7 = 0$ since the coefficient of the first term is 1.
Step 2: Instead, move -7 to the right side of the equation, so, $x^2 + 6x = 7$.

Step 3. $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 7 + \left(\frac{6}{2}\right)^2$

$x^2 + 6x + 3^2 = 7 + 3^2$

$x^2 + 6x + 9 = 7 + 9$

$(x + 3)(x + 3) = 16$

$(x + 3)^2 = 4^2$

$\sqrt{(x + 3)^2} = \pm\sqrt{4^2}$

$(x + 3) = \pm 4$ this implies that $x + 3 + (-3) = \pm 4 + (-3)$, that is, $x = \pm 4 - 3$. So, $x = +4 - 3 = 1$ and $x = -4 - 3 = -7$.

Therefore, the solution set of $x^2 + 6x - 7 = 0$ is $\{1, -7\}$.

5) The problem requires us to determine the number whose product of the digit is 12. We could write down as many two-digit numbers where the product of the digit is 12, for example, 26, 62, 34, and 43. However, an additional condition is whenever we add 9 to this two-digit number it will result to the interchanged of it digit places. So, let us consider the number 34. Now, if we add 9 to 34 it will give us 43, exactly what the second condition is asking. Therefore, the two-digit number is 34.

Now, let us have some collaborative work. Form a group of five (5) members and do the following as indicated. You will present your work in class.

Group Activity 1: TELL ME MY SOLUTIONS PLEASE!

Directions: Determine the root(s) of each of the following equations and answer the reflection questions below.

1) $x + 23 = 76$
2) $m + \frac{1}{2} = 3$
3) $8k + 56k^2 = 0$
4) $2x + 4x - 5x - 8 = 0$
6) $p^2 - 1 = 0$
7) $4x + 2x + 4 = 7$
8) $3d^2 - 15d = 0$
9) $(k + 3)^2 = 9$

5) $\frac{1}{2}x = 2$

10) $(t - \frac{1}{3})^2 = \frac{1}{4}$

Reflective Questions:

1. What can you notice about the solution(s) in Activity 1?
2. Which equations has only one solution? Why?
3. Which equations has two solutions? Why?
4. Can you name some mathematical properties applied in solving?

Group Activity 2: WRITE ME INTO MY EQUIVALENCE!

Directions: Express each of the following perfect square trinomial as a square of a binomial and answer the reflection questions below.

1) $x^2 + 6x + 9$
2) $w^2 + 14w + 49$
3) $v^2 - 22v + 121$
4) $y^2 + 20y + 100$
5) $b^2 - 10b + 25$
6) $64 - 16n + n^2$
7) $k^2 + k + \frac{1}{4}$
8) $\frac{4}{9} - \frac{4}{3}s + s^2$
9) $4x^2 + 4x + 1$
10) $25x^2 - 30x + 9$

Reflective Questions:

1. How will you describe a perfect square trinomial?
2. How did you express each trinomial into its equivalent square of binomial?
3. Can you name some mathematical properties applied in solving?
4. Compare your answer with your classmates. Do you have the same answer? If NOT, why?

Group Activity 3: LET'S DO THE COMPLETING THE SQUARE!

Directions: Find the solutions of each of the following quadratic equations using completing the square method:

1) $x^2 - 12x = -10$
2) $x^2 - 20x + 100 = 0$
3) $x^2 + 5x + 1 = 0$
4) $2x^2 - 3 + 5x = 0$
5) $x^2 + 4x = -4$
6) $6x^2 + x - 1 = 0$
7) $x^2 - 5x - 7 = 7$
8) $3x^2 - 3x - 6 = 0$
9) $-2 + x + x^2 = 0$
10) $2x^2 - x - 1 = 0$

Reflective Questions:

1) How do you find factoring in Activity 3? Can you describe your understanding on how you did the factoring method?

2) Discuss some of the strategies/techniques in factoring.
3) In what particular scenario you can apply the concept factoring?

Group Activity 4: LET'S APPLY!

1. Two pipes running together can fill a tank in 2.73 minutes. If one pipe takes a minute more than the other to fill the tank, find the time in which each pipe would fill the tank alone.
2. What are the integers if there are two consecutive positive integers whose sum of squares of the first and the product of the other is 46?
3. A computer manufacturing company would like to come up with a new laptop such that its monitor is 80 square inches smaller than the present ones. Suppose the length of the monitor of the larger computer is 5 inches longer than its width and the area of the smaller computer is 70 square inches. What are the dimensions of the monitor of the larger computer?

Reflective Questions:

1. How did you solve the word problem? Explain the steps you did.
2. Can you name some mathematical properties and principles applied in getting the factors of expressions? How did you apply it?
3. Give your own example where the concept of factoring is applied.

EXERCISES

A. Computation. Directions: Find the solutions of each of the following quadratic equations using completing the square method.

1) $x^2 + 4x = 6$
2) $x^2 - 2x - 3 = 0$
3) $x^2 + 7x - 30 = 0$
4) $-3x^2 + 2 = 5x$
5) $6x^2 - x - 1 = 0$
6) $12x^2 + 2x = 0$
7) $x^2 - 125 = 0$
8) $4x^2 - 16 = 0$
9) $5x^2 - 12 = 4$
10) $x^2 - 14x + 49 = 0$

B. Word Problem. Solve each of the following.

1. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

   Two years ago, Raj's age was three times the square of his younger brother's age. Three years hence his age will be four times his younger brother's age. What are their ages?

LET'S WORK THIS OUT

WORKSHEET NO. 4

QUADRATIC EQUATIONS: COMPLETING THE SQUARE

Direction: Find the solutions of each of the following quadratic equations using completing the square method.

1. $x^2 - 10x + 21 = 0$	2. $k^2 + 3k - 9 = 0$
3. $2m^2 + 32m + 88 = 0$	4. $0.1w^2 - 0.2w - 0.5 = 0$

Let's Reflect:

1. How do you find answering this worksheet? (*Please narrate your experience in one sentence.)*
2. Which part of the activity you find it challenging? (*Please specify by writing a sentence the challenges and difficulties encountered*.)

SUMMARY

- A quadratic equation can also be solved using the completing the square method.
- Completing the Square is a method used to solve a quadratic equation by changing the form of the equation so that the left side is a perfect square trinomial .
- Take note of this perfect square trinomials which can be used to easily determine the roots by following the pattern.
  - $a^2 + 2ab + b^2 = (a + b)^2$
  - $a^2 - 2ab + b^2 = (a - b)^2$

After completing this module, you are task to go online to take the posttest using your *GoConqr* account. Good luck!