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MATH 11N: MATHEMATICS IN THE MODERN WORLD

**Module 3**  
**QUADRATIC EQUATIONS**  
*(3.5. Solution of Quadratic Equation  
using Quadratic Formula)*

By

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## Solving Quadratic Equation using Quadratic Formula

Hi, how are you? This time, you will be introduced with the concepts of quadratic equation and solving using the Quadratic Formula. Before we start, let us consider first the following learning objectives.

### Learning Objective

After studying this lesson, you are expected to:

- solve quadratic equation using the quadratic formula.

### Let's Go Online!

I want you to visit the following links and study the materials.

1. <https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/a/quadratic-formula-explained-article>. This link presents a lesson video that explains the Quadratic Formula. Watch the video for further understanding.
2. <https://www.mathsisfun.com/algebra/quadratic-equation-real-world.html>. This link provides clear explanations with several applications of the concept. Exercises are also provided for mastery of the skill.
3. <https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/e/quadratic-equation>. This link provides practice problems which will help you strengthen your skill in solving quadratic equation by completing the square. Try solving each item. A complete solution was also provided for reference.
4. <https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/v/quadratic-formula-3>. This link provides a video lesson in solving quadratic equations using the Quadratic formula. Watch the video and take note some important concepts, including things you want to clarify.
5. <http://jwilson.coe.uga.edu/EMAT7050/Students/Wilson/Quad.htm>. This link presents an article by Jim Wilson of The University of Georgia. The discussion is centered on the teaching the derivation of the Quadratic formula. Read and study the material.

After watching the videos and studying the materials online, use your *GoConqr* account and answer the pre-assessment activity.

How do you find our lesson online? Is there anything you'd like to clarify about the presentation? Do you have listed all your concerns and clarifications? Let us discuss it now.

## DISCUSSIONS AND ACTIVITIES (Face-to-Face)

The simplest way to solve " $ax^2 + bx + c = 0$ " for the value of  $x$  is to factor the quadratic equation, set each factor equal to zero, and then solve each factor. But sometimes the quadratic equation is too messy, or it doesn't factor at all, or you just don't feel like factoring. While factoring may not always be successful, the *Quadratic Formula* can always find the solution.

The Quadratic Formula uses the coefficients " $a$ ", " $b$ ", and " $c$ " from " $ax^2 + bx + c$ ", where " $a$ ", " $b$ ", and " $c$ " are just numbers; they are the "numerical coefficients" of the quadratic equation they've given you to solve.

In this lesson, you will learn how to solve quadratic equation using the quadratic formula.

### Pre-requisites

You will need the following pre-requisite skills in this lesson:

- ✓ The concept of substitution;
- ✓ Writing quadratic equation in its standard form; and
- ✓ Rules/Properties of Radicals.

The Quadratic Formula is derived from the process of completing the square, and is formally stated below:

The Quadratic Formula: For  $ax^2 + bx + c = 0$  the values of  $x$  which are the solutions of the equation are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where  $a, b, c$  are numerical coefficients of the first, second and third terms in the given quadratic equation.

Before we proceed to solving quadratic equation using the formula, let us derive first the Quadratic Formula from the process of Completing the Square. Basically, the general form of the quadratic equation is given as  $ax^2 + bx + c = 0$ . So, by addition property of equality applied for the constant (the 3<sup>rd</sup> term) we have  $ax^2 + bx = -c$ . Moreover, the leading term its numerical coefficient will be divided across the equation, that is,  $x^2 + \frac{b}{a}x = -\frac{c}{a}$ . Now, we take half of the numerical coefficient of  $x$  which is  $+\frac{b}{a}$

(or simply multiply  $\frac{1}{2}$  to  $+\frac{b}{a}$ ) which will become  $+\frac{b}{2a}$  and get it square. Further, add

$\left(+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$  on the both sides of the equation. So,

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now, that you know already how the Quadratic Formula is derived from Completing the Square Method. You are now introduced with the steps in using the Quadratic Formula to facilitate in solving quadratic equation. The following are the steps in solving quadratic equation using Quadratic formula:

#### **STEPS in Solving Quadratic Equation using Quadratic Formula:**

**Step 1:** Write the quadratic equation in standard form.

**Step 2:** Substitute the values of a, b, and c into the formula.

**Step 3:** Evaluate to get the solutions. Leave your answer in simplest form.

## Examples

Directions: Solve the discriminant for each of the following quadratic equation.

1)  $3x^2 - 2x = 5$

3)  $4x^2 + 2x - 5 = 0$

2)  $x^2 - 3x + 1 = 0$

4)  $x^2 + 6x - 7 = 0$

- 5) A five-hour cruise goes 17 km up stream and then back again. The river has a current of 3 km an hour. What is the boat's speed and how long was the upstream journey?

### Solutions:

1)  $3x^2 - 2x = 5$

Step 1: Write  $3x^2 - 2x = 5$  in standard form. Hence,  $3x^2 - 2x - 5 = 0$ .

Step 2: Now, from Step 1, the numerical coefficients are  $a = 3, b = -2, c = -5$ .

$$\text{So, } x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-5)}}{2(3)}.$$

Step 3.  $x = \frac{2 \pm \sqrt{4+60}}{6} = \frac{2 \pm \sqrt{64}}{6} = \frac{2 \pm 8}{6}$ . Thus,  $x_1 = \frac{2+8}{6} = \frac{10}{6} = \frac{5}{3}$  and  $x_2 = \frac{2-8}{6} = \frac{-6}{6} = -1$ .

Therefore, the solution set of  $3x^2 - 2x = 5$  is  $\left\{\frac{5}{3}, -1\right\}$ .

2)  $x^2 - 3x + 1 = 0$

Step 1:  $x^2 - 3x + 1 = 0$  is in standard form already.

Step 2: Now, from Step 1, the numerical coefficients are  $a = 1, b = -3, c = 1$ .

$$\text{So, } x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}.$$

Step 3.  $x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$ . Thus,  $x_1 = \frac{3+\sqrt{5}}{2}$  and  $x_2 = \frac{3-\sqrt{5}}{2}$ .

Therefore, the solution set of  $x^2 - 3x + 1 = 0$  is  $\left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$ .

3)  $4x^2 + 2x - 5 = 0$

Step 1:  $4x^2 + 2x - 5 = 0$  is in standard form already.

Step 2: Now, from Step 1, the numerical coefficients are  $a = 4, b = 2, c = -5$ .

$$\text{So, } x = \frac{-(2) \pm \sqrt{(2)^2 - 4(4)(-5)}}{2(4)}.$$

Step 3.  $x = \frac{-2 \pm \sqrt{4+80}}{8} = \frac{-2 \pm \sqrt{84}}{8} = \frac{-2 \pm 2\sqrt{21}}{8}$ . Thus,  $x_1 = \frac{-2+2\sqrt{21}}{8}$  and  $x_2 = \frac{-2-2\sqrt{21}}{8}$ .

Therefore, the solution set of  $4x^2 + 2x - 5 = 0$  is  $\left\{\frac{-1+\sqrt{21}}{4}, \frac{-1-\sqrt{21}}{4}\right\}$ .

4)  $x^2 + 6x - 7 = 0$

Step 1:  $x^2 + 6x - 7 = 0$  is in standard form already.

Step 2: Now, from Step 1, the numerical coefficients are  $a = 1, b = 6, c = -7$ .

$$\text{So, } x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-7)}}{2(1)}.$$

Step 3.  $x = \frac{-(6) \pm \sqrt{36+28}}{2(1)} = \frac{-6 \pm \sqrt{64}}{2} = \frac{-6 \pm 8}{2}$ . Thus,  $x_1 = \frac{-6+8}{2} = \frac{2}{2} = 1$  and  $x_2 = \frac{-6-8}{2} = \frac{-14}{2} = -7$ .

Therefore, the solution set of  $x^2 + 6x - 7 = 0$  is  $\{1, -7\}$ .

- 5) There are two speeds we have to consider: the speed the boat makes in the water, and the speed relative to the land. Let  $x$  be the boat's speed in the water (in km/hr) and  $y$  be the speed relative to the land (in km/hr). Because the river flows downstream at 3 km/hr, we have

$$y = x - 3 \text{ is the speed reduced by 3 km/hr when going upstream}$$

$$y = x + 3 \text{ is the speed increased by 3 km/hr when going downstream}$$

So, from these given we can now use the formula time (t) = distance (d)/ speed (s) (this was basically derived from the distance formula). Now,

$$\text{Total time} = \text{time upstream} + \text{time downstream} = 5 \text{ hours}$$

$$\text{Total time} = \frac{17}{x-3} + \frac{17}{x+3} = 5. \text{ Now, let's get rid of the fractions by}$$

multiplying the whole equation by  $(x-3)(x+3)$ . We have,

$$17(x+3) + 17(x-3) = 5(x-3)(x+3). \text{ Further,}$$

$$17x + 51 + 17x - 51 = 5(x^2 - 9)$$

$$17x + 17x + 51 - 51 = 5x^2 - 45$$

$$34x = 5x^2 - 45$$

$$5x^2 - 34x - 45 = 0$$

Now, apply the Quadratic Formula,  $a = 5, b = -34, c = -45$  we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-34) \pm \sqrt{(-34)^2 - 4(5)(-45)}}{2(5)}$$

$$= \frac{34 \pm \sqrt{1156 + 900}}{10}$$

$$= \frac{34 \pm \sqrt{2056}}{10}$$

$$= \frac{34 \pm 2\sqrt{514}}{10}$$

$$= \frac{34 \pm 45.3}{10}$$

$$\frac{34+45.34}{10} = \frac{79.3}{10} = 7.93 \text{ and } \frac{34-45.34}{10} = \frac{-11.3}{10} = -1.13$$

So, the values of x are  $7.93 \approx 8$  and  $-1.13$ . However,  $-1.13$  is unrealistic. To answer the question in the problem, we have

Answer: Boat's Speed = 8 km/hr

And so, the upstream journey =  $\frac{17}{8-3} = \frac{17}{5} = 3.4$  hours

And so the downstream journey =  $\frac{17}{8+3} = \frac{17}{11} = 1.5$  hours

Now, let us have some collaborative work. Form a group of five (5) members and do the following as indicated.

### Group Activity 1: LET US RECAP!

Directions: Determine the root(s) of each of the following equations using any method discussed and answer the reflection questions below.

1)  $x^2 - 4x - 8 = 0$

6)  $3m^2 - 5m = 12$

2)  $x^2 + x - 3 = 0$

7)  $4 = w^2 - 4w$

3)  $k^2 - 6k = 10$

8)  $x^2 = -\frac{1}{3}x - 8$

4)  $h^2 + 4h = 3$

9)  $3x^2 + 4x + 5 = 0$

5)  $2x^2 - 5x - \frac{1}{2} = 0$

10)  $5x^2 - 25x - 10 = 0$

### Reflective Questions:

1. What can you say about finding the solutions in Activity 1?
2. Can you describe a pattern in solving? Please specify.
3. How will you relate this concept on completing the square to actual situation?

### Group Activity 2: RATIONAL OR IRRATIONAL?

Directions: Solve each of the following and identify whether its solution is rational or irrational. In the case of neither, justify. Answer the reflection questions below.

1)  $\sqrt{121}$

6)  $\sqrt{-9}$

2)  $\sqrt{0.25}$

7)  $\sqrt{648}$

3)  $\sqrt{120}$

8)  $\sqrt{\frac{1}{2} + \frac{1}{2} - 0}$

4)  $\sqrt{6}$

9)  $\sqrt{2^2 \cdot 3^2 \cdot 4^2}$

5)  $\sqrt{12}$

10)  $\sqrt{15129}$

### Reflective Questions:

1. How do you think it is rational or irrational?
2. Which of these items has a rational solution?
3. Which of these items has an irrational solution?
4. What do you think is the importance of this skill in solving quadratic equation using the Quadratic formula?

### Group Activity 3: LET US SOLVE!

Directions: Determine the solutions of each of the following quadratic equations using quadratic formula.

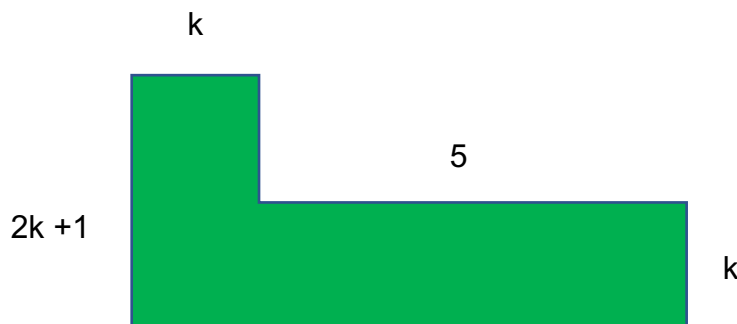
- |                           |                        |
|---------------------------|------------------------|
| 1) $x^2 - 12x = -10$      | 6) $6x^2 + x - 1 = 0$  |
| 2) $x^2 + 3x + 4 = 1 + x$ | 7) $x^2 - 5x - 7 = 7$  |
| 3) $x^2 + 5x + 1 = 0$     | 8) $3x^2 - 3x - 6 = 0$ |
| 4) $2x^2 - 3 + 5x = 0$    | 9) $-2 + x + x^2 = 0$  |
| 5) $x^2 + 4x = -4$        | 10) $2x^2 - x - 1 = 0$ |

### Reflective Questions:

- 1) How do you find factoring in Activity 3? Can you describe your understanding on how you did the computation?
- 2) Discuss some mathematical principles and properties that are applied.
- 3) Construct your own situation applying the concept of solving quadratic equation using Quadratic Formula.

### Group Activity 4: LET US APPLY!

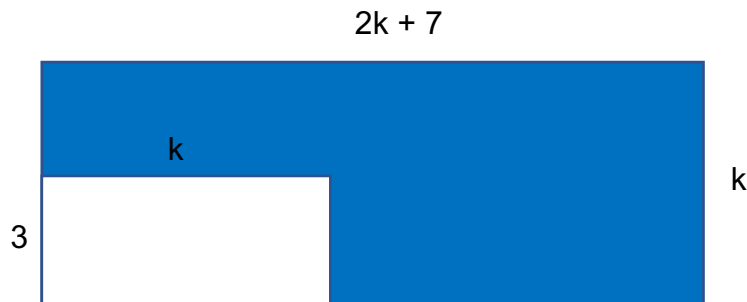
1. The length of a rectangle is 5 m more than its width and the area is 50 square meters. What is the length, width and the perimeter of the rectangle?
2. Consider the figure below:



If the area of the figure above is 80 square meters, what the value of k?



3. The following rectangular shape in which a smaller rectangular part (white color) is to be removed. If the remaining area of the larger rectangle (blue color) is 42 sq. cm., what is the value of k?



### Reflective Questions:

1. How did you solve the word problem? What strategy did you apply?
2. Can you name some mathematical properties and principles applied in getting the factors of expressions? How did you apply it?
3. What do you think is the importance of studying quadratic equations?

## EXERCISES

**A. Computations.** Directions: Determine the solutions of each of the following quadratic equations using quadratic formula.

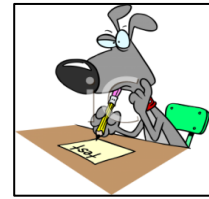
- |                        |                          |
|------------------------|--------------------------|
| 1) $x^2 + 5x = 6$      | 6) $12x^2 + 2x = 0$      |
| 2) $5x^2 - 12 = 48$    | 7) $x^2 - 125 = 0$       |
| 3) $x^2 + 7x - 30 = 0$ | 8) $4x^2 - 16 = 0$       |
| 4) $-3x^2 + 2 = 5x$    | 9) $x^2 - 2x - 3 = 0$    |
| 5) $6x^2 - x - 1 = 0$  | 10) $x^2 - 14x + 49 = 0$ |

**B. Word Problem.** Solve each of the following.

1. The shortest side of a right triangle is 6m shorter than its longest side which is the hypotenuse. The difference in length of other two sides is 3m. If the shortest side is  $a - 3$ , prove that  $2a^2 = 12a$ . What is the value of  $a$ ?
2. The base length of a triangle is 2 inches more than its height. The area is 24 square inches. What is the length of the hypotenuse and the perimeter of the triangle?

## LET'S WORK THIS OUT

### WORKSHEET NO. 5 QUADRATIC EQUATIONS: QUADRATIC FORMULA



**Direction:** Determine the solutions of each of the following quadratic equations using quadratic formula.

1.  $x^2 + 3x + 2 = 0$

2.  $3k^2 - 5k = -2$

3.  $m^2 + m - 4 = 0$

4.  $\frac{1}{2}w^2 - \frac{1}{3}w + \frac{1}{6} = 0$

**Let's Reflect:**

1. How do you find answering this worksheet? *(Please narrate your experience in one sentence.)*
2. Which part of the activity you find it challenging? *(Please specify by writing a sentence the challenges and difficulties encountered.)*

## SUMMARY

- Quadratic formula can be used in all forms of the quadratic equation.
- In order for the Quadratic Formula to work, you must arrange and write your equation in the form "(quadratic)" = 0.
- It is important that your quadratic equations are properly labelled as to the value of the variables  $a$ ,  $b$  and  $c$ .
- The denominator  $2a$  of the formula is underneath everything above, not only the expression with radical sign.
- Do not forget that there is a negative sign in "b" and is separated by " $\pm$ " between the expression with radical sign.

After completing this module, you are task to go online to take the posttest using your GoConqr account.