DEPARTMENT OF TEACHER

# MATH 11N: MATHEMATICS IN THE MODERN WORLD Module 3 QUADRATIC EQUATIONS 

## (3.2. Solution by Square Root Method)

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## Quadratic Equation: Solutions by Extracting the Square Roots

Hi, today you will be learning about Quadratic Equation and its solution using extracting the square root. Before we start, let us consider first the following learning goal.

## Learning Objective

After studying this lesson, you are expected to

- find the solutions of a quadratic equation using extracting the square roots.


## Let's Go Online!

I want you to visit the following links and study the materials.

1. https://www.onlinemathlearning.com/square-root-word-problems-2.html.This link provides a video presentation of word problems involving square root methods. Watch and study the material for further understanding.
2. https://www.khanacademy.org/math/algebra/ x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:untitled 1082/e/solving quadratics by taking the square root. This link provides practice exercises to help you in in solving quadratic equation by extracting the roots.
3. https://2012books.lardbucket.org/books/advanced-algebra/s09-01-extracting-square-roots-and-co.html. This link provides clear discussions with examples of solving quadratic equations using the Square Root Method. After reading and studying the material, answer the exercise on Part A. Write your answer in a short bonder paper.

After watching the videos and studying the materials online, use your GoConqr account and answer the pre-assessment activity.

How do you find our lesson online? Is there anything you'd like to clarify about the presentation? Do you have listed all your concerns and clarifications? Let us discuss it now.

## DISCUSSIONS AND ACITVITIDS (Face-to-Face)

In this lesson, you will learn to define and solve quadratic equation and find its solutions by extracting the square roots.

Basically, the standard quadratic equation in one variable is written as the sum of three terms set equal to zero. Simply, the three terms include one that has an $x^{2}$, one has an $x$, and one term is "by itself" with no $x^{2}$ or $x$.

Hence, the standardized form of a quadratic equation is $a x^{2}+b x+c=0$, where "a" does not equal 0 . Note that if $a=0$, the $x^{2}$ term would disappear and we would have a linear equation!

What about a quadratic equation? Well, if we look at the simplest case when a $=1$, and $b=c=0$, we get the equation $y=1 x^{2}$ or $y=x^{2}$. Since the variable involve has an exponent " 2 " such expression is considered quadratic. However, in this module our focus will be on solving quadratic equation in one variable

## Pre-requisites Skills

The following prior knowledge and skills are needed in this lesson:
$\checkmark$ The concept of rational and irrational numbers;
$\checkmark$ The concept and skill in getting the square root of numbers;
$\checkmark$ Product of sum and difference;
$\checkmark$ Properties of equality; and
$\checkmark$ Some properties of radical may be applied as well, especially for numerical coefficient.

Basically, polynomials are classified based on the highest degree of the variable involved. For example, $x+5$ is a polynomial of the first degree, $x^{2}-5 x+6$ is an example of a polynomial of the second degree, $1+x^{3}$ is a third degree polynomial.

The presence of equal sign together with the polynomial of any degree is called an equation. Equation is a mathematical operation which compares two quantities. Say for instance, $x+5=0$ and is called linear equation. Another is, $x^{2}-5 x+6=0$ which is a quadratic equation.

Now, let us have some collaborative work. Form a group of five (5) members and do the following as indicated.

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## Group Activity 1: TELL ME IF I'M RATIONAL OR IRRATIONAL!

Directions: Determine the product for each of the following pairs of expressions and answer the questions below.

1) $(x+4)(x-4)$
2) $(x-13)(x+13)$
3) $x(2)(x-1)$

$$
\text { 7) }(x-1)(x-2)
$$

3) $x(x+4)$
4) $\left(m^{2}+1\right)(m-1)$
5) $(9-x)(9+x)$
6) $3\left(w^{2}+3 w+5\right)$
7) $\left(k^{2}+1\right) k$
8) $(2 a+1)(3 a-1)$

## Reflective Questions:

1. How did you find the product?
2. Can you name some mathematical properties and principles applied in multiplying the pairs of expressions? How did you apply it?
3. In determining the product of each pair, what have you noticed? Are they all polynomials? Why or why not?

## Group Activity 2: QUADRATIC OR NOT QUADRATIC EQUATION?

Directions: Identify whether or not the following are quadratic equations. Answer the questions below.

1. $x^{2}=25$
$6 \quad 3 x^{3}-2=46 \mathrm{x}$
2. $n=24$
$7 \quad 5 x^{2}-6=39$
3. $x^{2}-1 / 2=0 s$
$8-5 x^{2}+32=-3 x^{2}$
4. $4 x^{2}-100=0$
$9(x-9)^{2}=64$
5. $x^{2}+4=x^{2}$
$10 x(x-4)^{2}+25 x=650$

## Reflective Questions:

1. What can you say about the activity?
2. Were you able to identify which of the equations are quadratic?
3. Can you share your thought on what are the conditions to say that a certain equation is quadratic or not a quadratic?

Now, as defined, a quadratic equation is any equation of the form $\mathrm{ax}^{2}+\mathrm{bx}+$ $\mathrm{c}=0$ where $\mathrm{a}, \mathrm{b}$ and c are real numbers and a is not zero. There are several methods to determine the solution of the quadratic equation. One of the methods is the so called Extracting the Square Roots or simply Square Root Method, which is only applicable in an incomplete quadratic equation. That is, the equation $a x^{2}-c=0$ is first solved for $x^{2}$ by adding $c$ to both members and then dividing both sides of the resulting
equation by $a$, the coefficient of $x^{2}$, resulting to $x^{2}=\frac{c}{a}$. Upon extracting the square roots of each member, we obtain,

$$
x= \pm \sqrt{\frac{c}{a}}
$$

Thus, $a x^{2}-c=0$ has two roots. That is, $x=+\sqrt{\frac{c}{a}}$ and $x=-\sqrt{\frac{c}{a}}$.

## STEPS in Extracting Square Root:

To solve a quadratic equation $a x^{2}-c=0$ using extracting the square root, the following steps might help you in solving:

1) Add to both sides of the equation by the additive inverse of $c$.
2) Divide both sides of the equation by $a$ or you may multiply each side of the resulting equation by the multiplicative inverse of $a$, i.e., $\frac{1}{a}$.
3) Take the square root of each side.
4) Simplify the result, if possible.

## Illustrative Examples

Directions: Solve each of the following quadratic equations by extracting the square.

1) $3 x^{2}-27=0$
2) $2 x^{2}+4=36$
3) $(x-3)^{2}=49$
4) $(x-2)^{2}-9=40$
5) A gardener is landscaping in a certain park and wanted to form a circular formation of plants. If the gardener is given with an area of $100 \pi$ units, find the radius of the circular area.

## Solutions:

1) $3 x^{2}-27=0$

Step 1: $3 x^{2}-27+(27)=0+27$ $3 x^{2}=27$
Step 2: $\frac{3 x^{2}}{3}=\frac{27}{3}$

$$
x^{2}=9
$$

Step 3. $\sqrt{x^{2}}= \pm \sqrt{9}$
Step 4: $x= \pm 3$
2) $2 x^{2}+4=36$

Step 1: $2 x^{2}+4+(-4)=36+(-4)$
roots: $x= \pm 3$
By adding 27 to both sides of the equation
Dividing both sides of the equation by 3.

Extracting square root.
Thus, $x=3$ and $x=-3$.
roots: $x= \pm 4$
By adding -4 to both sides of the equation.
$2 x^{2}=32$
$2 x^{2}$
Step 2: $\frac{2 x^{2}}{2}=\frac{32}{2} \quad$ Dividing both sides of the equation by 2.

$$
x^{2}=16
$$

Step 3. $\sqrt{x^{2}}= \pm \sqrt{16}$
Extracting square root.
Step 4: $x= \pm 4$
Thus, $x=4$ and $x=-4$.
3) $(x-3)^{2}=49$

Step 1 and 2:
Step 3. $(x-3)^{2}=49$

roots: $x=\{10,-4\}$
Done.
Extracting square root.
Step 4: $x-3= \pm 7$

$$
x= \pm 7+3
$$

This implied that $x=+7+3=10$ and $x=-7+3=-4$. Thus, $x=10,-4$.
4) $(x-2)^{2}-9=40$
roots: $x=\{9,-5\}$
By adding 9 to both sides of the equation
$(x-2)^{2}=49$
Step 2: Done, since the value of a here is 1 . Any number multiply/divide by 1 is the number itself.
Step 3. $\sqrt{(x-2)^{2}}= \pm \sqrt{49}$
Extracting square root.
$x-2= \pm 7$
Applying Step 1, we have
$x-2+2= \pm 7+2$.
So, $x=+7+2=9$ and
$x=-7+2=-5$.
Hence, $x=\{9,-5\}$.
5) The formula for area of a circle is $A=r^{2} \pi$. So,

$$
\begin{aligned}
& \text { A= } r^{2} \pi \\
& 100 \pi=r^{2} \pi \\
& 100=r^{2} \text { implies } r=10 \text {. Take note that, we only } \mathrm{ne} \\
& \text { we are dealing with unit or dimension. } \\
& \text { Thus, the radius of the circular area is } 10 \text { units. } \\
& \text { Group Activity 3: CAN YOU EXTRACT ME? }
\end{aligned}
$$

$100=r^{2}$ implies $r=10$. Take note that, we only need the positive root here because

Directions: Determine the roots of each of the following quadratic equations using extracting the square roots.

1. $x^{2}=25$
2. $6 x^{2}=24$
3. $x^{2}-100=0$
4. $4 x^{2}+100=0$
5. $x^{2}+4=53$
6. $3 x^{2}-2=46$
7. $5 x^{2}-6=39$
8. $-5 x^{2}+36=-3 x^{2}$
9. $(x-9)^{2}=64$
10. $(x-4)^{2}+25=650$

## Reflective Questions:

1) How do you find determining the roots of the quadratic equation by Square Roots Method?
2) Discuss the steps in solving quadratic equation using extracting the square roots.
3) Can you name a situation/s in which the concept of quadratic equations is applied?

## Group Activity 4: LET'S APPLY!

Directions: Solve the following word problem.

1. If the square of a certain number is doubled yield one. Find the number.
2. What must be multiplied to $(x-7)$ in order to have the quadratic equation of the form $x^{2}=49$.
3. If the side length of a square is increased by 5 , its area increases to 85 . What is the side length of the square?

## Reflective Questions:

1. How do you find solving word problem involving quadratic equation?
2. Cite a situation in which solving quadratic equation by extracting the roots is applicable.

## EXERCISES

A. Computation. Directions: Determine the roots of each of the following quadratic equations using extracting the square roots.

1. $x^{2}=121$
2. $5 x^{2}=125$
3. $x^{2}-81=0$
4. $4 x^{2}-16=0$
5. $x^{2}+4=20$
6. $2 x^{2}-2=16$
7. $5 x^{2}-12=48$
8. $-5 x^{2}+12=-2 x^{2}$
9. $(x-1)^{2}=4$
10. $(x-7)^{2}-9=27$
B. Word Problem. Solve each of the following.
11. The area of a square is 289 square units. What is the side length of the square?
12. The length of the side of a square is increased by 3 , its area increases 54 . What is the side length of the square?

## LETYS WORE THIS OUT

## WORKSHEET NO. 2 QUADRATIC EQUATIONS: EXTRACTING THE SQUARE ROOTS



Direction: Determine the roots of each of the following quadratic equations using the square root property. Please write your solution on the space provided.

| 1. $x^{2}-5=116$ |  |
| :--- | :--- |
|  | 2. $(k+4)^{2}=25$ |
| 3. $5 m^{2}-14=-9$ |  |
|  | $4 .-3+13 w^{2}=4209$ |

## Let's Reflect:

1. How do you find answering this worksheet? (Please narrate your experience in one sentence.)
2. Which of the given quadratic equations you find it challenging to solve? Why? (Please specify by writing a sentence the challenges and difficulties encountered.)

## SUMMART

- Solve quadratic equations of the form $a x^{2}-c=0$ by extracting the square roots.
- Extracting roots requires isolating the square and application of square root property. Be sure to simplify each radical expression after applying the square root property
- Do not forget to include " $\pm$ " in your final answer.

After completing this module, you are task to go online to take the posttest using your GoConqr account. Good luck!

