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MATH 11N: MATHEMATICS IN THE MODERN WORLD

Module 3
QUADRATIC EQUATIONS
*(3.1. Definition and Characteristics of the
Roots of a Quadratic Equation)*

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Definition and Characteristics of the Roots of a Quadratic Equation

Hi, today you will be learning about Quadratic Equation, its definition and nature of its roots. Before we start, let us consider first the following learning objectives.

Learning Objectives

After studying this lesson, you are expected to

- illustrate quadratic equations;
- compute for the discriminant of the quadratic equation; and
- describe and characterize the roots of a quadratic equation using the discriminant.

Let's Go Online!

I want you to visit the following links and study the materials.

1. <https://www.toppr.com/guides/maths/quadratic-equations/nature-of-roots/>; and <https://www.onlinemath4all.com/nature-of-the-roots-of-a-quadratic-equation.html>. This link provides a comprehensive discussion of the nature of Roots of quadratic equation. Study the material for understanding.
2. https://www.google.com.ph/search?safe=strict&client=safari&channel=ipad_bm&source=hp&ei=HiVqXtDvK82o-QaHjpPoAQ&q=finding+discriminant+quadratic+equation&og=finding+discriminant. This link provides a video presentation that will help you in solving the discriminant of a quadratic equation.
3. <https://www.math-only-math.com/worksheet-on-nature-of-the-roots-of-a-quadratic-equation.html>. This link provides exercises on determining the nature and characteristics of the roots of a quadratic equation. Answer and study this material.

After watching the videos and studying the materials online, use your *GoConqr* account and answer the pre-assessment activity.

How do you find our lesson online? Is there anything you'd like to clarify about the presentation? Do you have listed all your concerns and clarifications? Let us discuss it now.

DISCUSSIONS AND ACTIVITIES (Face-to-Face)

The term *quadratic* comes from the word *quadrate* meaning *square* or *rectangular*. Similarly, one of the definitions of the term quadratic is a square. In an algebraic sense, the definition of something quadratic involves the square and no higher power of an unknown quantity but “2” or second degree. So, in this worksheet, we will be working with quadratic equations which mean that the highest degree we'll be encountering is a square or 2.

DEFINITION:

A **quadratic equation** in a single variable is any equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b and c are real numbers and $a \neq 0$.

It is important to know and familiarize the characteristics of the roots of a given quadratic equation. In some way, this might help you to understand the nature of the solutions of a quadratic equation.

In this module, you will learn how to compute for the discriminant of a quadratic equation and identify the characteristics of its roots. This module will focus on the discussions of quadratic equations particularly on identifying the nature or characteristics of the roots of the quadratic equation. Before we proceed to determine the said identification, please remember the following prerequisite.

Pre-requisites

The prior knowledge and skill for this lesson are the following:

- ✓ The skill in solving quadratic equation using the quadratic formula particularly on the expression under the radical sign.

In the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression $b^2 - 4ac$ is called the **discriminant**. It is used to determine the nature of the roots of a quadratic equation. The following table shows the nature of the roots of a quadratic equation.

Characteristics of the Roots of a Quadratic Equation

When the value of the discriminant, $b^2 - 4ac$ is	The roots of $ax^2 + bx + c = 0$ are
Zero	Rational and identical
Positive and perfect square	Rational and distinct
Positive but not perfect square	Irrational and distinct
Negative	Complex conjugates

Illustrative Examples

Directions: Compute the discriminant for each of the following quadratic equations and identify the characteristics of its roots.

1) $3x^2 - 2x = 5$

3) $4x^2 + 2x - 5 = 0$

2) $x^2 - 3x + 1 = 0$

4) $x^2 + 6x - 7 = 0$

Solutions:

1) $3x^2 - 2x = 5$

Step 1: Write $3x^2 - 2x = 5$ in standard form. Hence, $3x^2 - 2x - 5 = 0$.

Step 2: Now, from Step 1, the numerical coefficients are $a = 3, b = -2, c = -5$.

Step 3. $b^2 - 4ac = (-2)^2 - 4(3)(-5) = 4 + 60 = 64$.

So, since the discriminant is greater than 0 then this quadratic equation will have two real roots.

2) $x^2 - 3x + 1 = 0$

Step 1: $x^2 - 3x + 1 = 0$ is in standard form already.

Step 2: Now, from Step 1, the numerical coefficients are $a = 1, b = -3, c = 1$.

Step 3. $b^2 - 4ac = (-3)^2 - 4(1)(1) = 9 - 4 = 5$.

The value of the discriminant is 5 and is positive but not a perfect square. Thus, its roots are irrational and distinct.

You can observe that it is more convenient way to describe the characteristics of the roots of quadratic equation simply by using the expression $b^2 - 4ac$. Hence, no need to solve the equation, you only need to compute for the discriminant.

3) $4x^2 + 2x - 5 = 0$

Since $4x^2 + 2x - 5 = 0$ is in standard form already, we need to set the numerical coefficients as $a = 4, b = 2, c = -5$ based from the given quadratic equation. Now, using the expression $b^2 - 4ac$, the value of the discriminant will be $(2)^2 - 4(4)(-5) = 4 + 80 = 84$. Therefore, the solution set of $4x^2 + 2x - 5 = 0$ is positive and but not a perfect square. Thus, its roots are irrational and distinct.

4) $x^2 + 6x - 7 = 0$

The given quadratic equation $x^2 + 6x - 7 = 0$ is already in standard form. Now, we write the numerical coefficients such as $a = 1, b = 6, c = -7$ and substitute to the expression $b^2 - 4ac$. So, we now have $(6)^2 - 4(1)(-7) = 36 + 28 = 64$. Hence, the discriminant of the quadratic equation $x^2 + 6x - 7 = 0$ is positive and a perfect square. Thus, its roots are rational and distinct.

Now, let us have some collaborative work. Form a group of five (5) members and do the following as indicated.

Group Activity 1: TELL ME IF I'M RATIONAL OR IRRATIONAL!

Directions: Solve each of the following and identify whether its solution is rational or irrational and answer the reflection questions below.

- | | |
|------------------|---|
| 1) $\sqrt{121}$ | 6) $\sqrt{15}$ |
| 2) $\sqrt{0.25}$ | 7) $\sqrt{648}$ |
| 3) $\sqrt{120}$ | 8) $\sqrt{\frac{1}{2} + \frac{1}{2} - 0}$ |
| 4) $\sqrt{6}$ | 9) $\sqrt{2^2 \cdot 3^2 \cdot 4^2}$ |
| 5) $\sqrt{12}$ | 10) $\sqrt{15129}$ |

Reflective Questions:

1. How do you find the activity?
2. Which of these items has a rational solution?
3. Which of these items has an irrational solution?

Group Activity 2: QUADRATIC OR NOT QUADRATIC EQUATION?

Directions: Identify each of the following is quadratic or not a quadratic equation. Also, answer the reflective questions.

- | | |
|----------------------------------|------------------------------|
| 1) $x^2 - 4x - 8 = 0$ | 6) $3m^2 - 5m = 12$ |
| 2) $x^2 + x - 3 = 0$ | 7) $4 = w^2 - 4w$ |
| 3) $k^2 - 6k = 10$ | 8) $x^2 = -\frac{1}{3}x - 8$ |
| 4) $h^2 + 4h = 3$ | 9) $3x^2 + 4x + 5 = 0$ |
| 5) $2x^2 - 5x - \frac{1}{2} = 0$ | 10) $5x^2 - 25x - 10 = 0$ |

Reflective Questions:

1. Define a quadratic equation.
2. Can you describe a pattern in identifying quadratic equation? Please specify.
3. How will you relate this concept on completing the square to actual situation?

Group Activity 3: WRITE ME IN MY STANDARD FORM!

Directions: Write the standard form for each quadratic equation below. Use the table to indicate the numerical coefficient for item, then answer the reflective questions.

- | | |
|---------------------------|------------------------|
| 1) $x^2 - 12x = -10$ | 6) $6x^2 + x - 1 = 0$ |
| 2) $x^2 + 3x + 4 = 1 + x$ | 7) $x^2 - 5x - 7 = 7$ |
| 3) $x^2 + 5x + 1 = 0$ | 8) $3x^2 - 3x - 6 = 0$ |
| 4) $2x^2 - 3 + 5x = 0$ | 9) $-2 + x + x^2 = 0$ |
| 5) $x^2 + 4x = -4$ | 10) $2x^2 - x - 1 = 0$ |

Item No.	Quadratic Equation	Standard Form	a	b	c
Example	$2x + x^2 - 3 = 0$	$x^2 + 2x - 3 = 0$	1	2	-3
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Reflective Questions:

1. How do you find factoring in Activity 3
2. How can you characterize a quadratic equation in terms of its roots?
3. Construct your own situation applying the concept.

Group Activity 4:

SOLVE THE DISCRIMINANT & TELL THE NATURE OF ITS ROOTS!

Directions: Compute the discriminant for each of the following quadratic equations and identify the nature/characteristics of its roots.

- | | |
|---------------------------|------------------------|
| 1) $x^2 - 12x = -10$ | 6) $6x^2 + x - 1 = 0$ |
| 2) $x^2 + 3x + 4 = 1 + x$ | 7) $x^2 - 5x - 7 = 7$ |
| 3) $x^2 + 5x + 1 = 0$ | 8) $3x^2 - 3x - 6 = 0$ |
| 4) $2x^2 - 3 + 5x = 0$ | 9) $-2 + x + x^2 = 0$ |
| 5) $x^2 + 4x = -4$ | 10) $2x^2 - x - 1 = 0$ |

Reflective Questions:

1. How do you compute for the discriminant? What do you think is the purpose of computing it?
2. How can you characterize a quadratic equation in terms of its roots?
3. What do you think is the importance of solving discriminant in a quadratic equation?
4. Construct your own situation applying the concept.

EXERCISES

Directions: Determine the value of the discriminants of each of the following quadratic equations then identify the nature of its roots.

1) $x^2 + 5x + 2 = 0$

6) $2x^2 - 3x + 6 = 0$

2) $5x^2 - 12 = 48$

7) $x^2 - 125 = 0$

3) $x^2 + 7x - 30 = 0$

8) $4x^2 - 16 = 0$

4) $-3x^2 + 2 = 5x$

9) $x^2 - 2x - 3 = 0$

5) $6x^2 - x - 1 = 0$

10) $x^2 - 14x + 49 = 0$

LET'S WORK THIS OUT

WORKSHEET NO. 1 QUADRATIC EQUATIONS: DISCRIMINANT AND THE CHARACTERISTICS OF ITS ROOTS



Direction: Compute for the discriminant of each of the following quadratic equations and identify the characteristics of its roots.

1. $x^2 - 5 = 116$

2. $(k + 4)^2 = 25$

3. $-3 + 13w^2 = 4209$

4. $5m^2 - 14 = -9$

1. $216x^2 - 27x = 0$

2. $k^2 - k - 72 = 0$

3. $12m^2 - 17m - 5 = 0$

4. $2w^2 - 12w = 2w + 60$

$$1. x^2 - 10x + 21 = 0$$

$$2. k^2 + 3k - 9 = 0$$

$$3. 2m^2 + 32m + 88 = 0$$

$$4. 0.1w^2 - 0.2w - 0.5 = 0$$

$$1. x^2 + 3x + 2 = 0$$

$$2. 3k^2 - 5k = -2$$

$$3. m^2 + m - 4 = 0$$

$$4. \frac{1}{2}w^2 - \frac{1}{3}w + \frac{1}{6} = 0$$

LET'S REFLECT:

1. Define quadratic equation. How will you characterize an equation that is quadratic?
(Please narrate your experience in one sentence.)

2. Which part of the activity you find it challenging? (Please specify by writing a sentence the challenges and difficulties encountered.)

SUMMARY

- Take note that in $ax^2 - c = 0$ or $x = \pm\sqrt{k}$ where $k = \frac{c}{a}$ using the square root method,
 - If $k > 0$, then $ax^2 - c = 0$ has two real solutions or roots
 - If $k = 0$, then $ax^2 - c = 0$ has one real solution or root
 - If $k < 0$, then $ax^2 - c = 0$ has no real solutions or roots
- It is possible that even if we do not solve the quadratic equation, its roots can be identified by simply using the discriminant $b^2 - 4ac$.

After completing this module, you are task to go online to take the posttest using your *GoConqr* account. Good luck!