MATH 11N: MATHEMATICS IN THE MODERN WORLD

Module 3

QUADRATIC EQUATIONS

(3.3. Solutions of Quadratic Equation by Factoring)

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Solving Quadratic Equation using Factoring Method

Dear students, today you will learn solving quadratic equation using another method called factoring method. Your knowledge on prime factorization can be applied here.

Learning Objective

After studying this module, you are expected to

- to solve quadratic equation using factoring method.

Let's Go Online!

I want you to visit the following links and study the materials.

1. https://www.purplemath.com/modules/factquad.htm. This link provides a comprehensive discussion on factoring quadratic equation.
2. https://www.mathsisfun.com/algebra/factoring-quadratics.html. This link provides illustrations of the different strategies in factoring. After studying the link, you are to take the practice exercises to help you in strengthen your skill in solving quadratic equation by factoring.
3. https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratics-multiplying-factoring/x2f8bb11595b61c86:factor-quadratics-intro/e/factoring_polynomials_1. This link provides practice exercises in solving quadratic equations using factoring. Take the exercises and write your answer in a short bonder paper.

After watching the videos and studying the materials online, use your *GoConqr* account and answer the pre-assessment activity.

How do you find our lesson online? Is there anything you'd like to clarify about the presentation? Do you have listed all your concerns and clarifications? Let us discuss it now.

DISCUSSIONS AND ACTIVITIES (Face-to-Face)

The word *factoring* refers to the art of writing algebraic expressions as a product of its factors. The process is actually division because when you multiply the divisor and the quotient, the product is the dividend.

In this lesson, you will learn how to solve quadratic equation using factoring method.

Pre-requisites

The prior knowledge and skills for this lesson are the following:

- ✓ The concept of prime factorization, special product and factoring;
- ✓ The concept and skill in getting the factors of numbers or expression; and
- ✓ The rule of addition and multiplication property of equality.

First, let's note that quadratic is another term for second degree polynomial. So, we know that the largest exponent in a quadratic polynomial is 2. The method factoring quadratic equations in standard form, $ax^2 + bx + c = 0$, can often be accomplished by finding two numbers that when we add them it will give us *b,* the numerical coefficient of the middle term; and when we multiply these two numbers will yield *c,* which is the third term or the constant in the quadratic equation.

Now, let us have some collaborative work. Form a group of five (5) members and do the following as indicated.

Activity 1: FIND ME GCF!

Directions: Determine the greatest common factor (GCF) for each of the following expressions and answer the reflection questions below.

1) $15,\ 35$
2) $16,\ 30$
3) $8,\ 56$
4) $100, 34,\ 215$
5) $85,\ 15,\ 120$
6) $p^2, 3p^3, 4p^4, 2p$
7) $4x,\ 2x + 4$
8) $3d^2,\ 15d$
9) $10k^3 + 5k,\ 45k$
10) $20m^2n,\ 5m,\ 25mn$

Reflective Questions:

1. What is GCF?
2. How did you find the GCF in Activity 1?
3. What mathematical concepts did you use in finding the GCF of the given items?
4. Is GCF a requirement in factoring method? Why?

Activity 2: WHAT IS COMMON OF US!

Directions: Determine the common factor for each of the following expressions and answer the reflection questions below.

1) $2x + 12$
2) $d + 4d - 3d$
3) $x^2 + 4x$
6) $m^2 + 3m^3 - 4m^4 + 5m^5$
7) $25, 150, 125, 35$
8) $3w^2 + 15$

4) $(m^2 + m + 2m)$

5) $(-3 - x)(3 + x)$

9) $k^3 + k$

10) $12m^2 + 6m$

Reflective Questions:
1. How did you find the common factor?
2. Can you name some mathematical properties and principles applied in getting the factors of expressions? How did you apply it?
3. In determining the factors of each pair, what have you noticed?

You learned in your basic algebra about the rule on multiplication of zero. This states that product of any number and zero is zero, that is, *a* • 0 = 0 where a is any real number except zero. This concept is applied when we factor out expression particularly on quadratic equations.

So, to solve quadratic equations by factoring, we use the ***Principle of Zero Products***. The zero product principle states that if the product of two factors is zero, then at least one of the factors must be zero. In symbols, we have

"*If* $ab = 0$ *then* $a = 0$ *or* $b = 0$."

So, since we are dealing with expressions whenever there is a polynomial set equal to a value (whether an integer or another polynomial), the result is an equation. For example, $2n + 12 = 10,\ m^2 + 4 = 0,\ d = 23,$ and $14t^2 = 28t.$

Hence from Activity 1 and 2, we can say that factoring is the process by which we go about determining what we are going to multiply to get the given quantity or finding a common factor of each term in a given expression. We do this all the time with numbers. The following are the steps in factoring:

STEPS in Factoring Quadratic Equation:

Step 1: Write the **equation** in the standard form, $ax^2 + bx + c = 0$.

Step 2: Use **factoring** strategies to factor the problem.

Step 3: Use the **Zero Product Principle** and set each factor containing a variable equal to zero. That is, if $ab = 0$, then either $a = 0$ or $b = 0$.

Step 4: Solve each factor that was set equal to zero and apply addition/multiplication property of equality leaving the variable x on one side and the constant value on the other side.

Illustrative Examples

Directions: Solve each of the following quadratic equations by factoring method.

1) $x^2 + 2x - 15 = 0$
2) $x^2 + 5x = -6$
3) $3x^2 - 8 = 10x$
4) $4x^2 + 10x - 6 = 0$
5) An arrow is projected straight up into the air with an initial velocity of 10 ft/s. At what times will the arrow be 25 ft above the ground? Use the equation height $(h) = 10t - t^2$ where h is the height (in feet), above the ground after t seconds.

Solutions:

1) $x^2 + 2x - 15 = 0 \quad \Rightarrow \quad (x + 5)(x - 3) = 0$

Step 1: $x^2 + 2x - 15 = 0$ is in standard form already.

Step 2: The last term is -15, so we will look for factors of -15 that when we get the sum will give us the middle term +2. The factors of -15 are $(-1)(15), (1)(-15), (-3)(5), \text{and } (3)(-5)$. However, it is the third pair that will satisfy the middle term.

Step 3. $(x + 5)(x - 3) = 0$ implies $(x + 5) = 0 \text{ or } (x - 3) = 0$ by Zero Product Principle.

Step 4: Further, $(x + 5) + (-5) = 0 + (-5)$. Hence, $x = -5$. And $(x - 3) + (+3) = 0 + (+3)$. Thus, $x = +3$.

Therefore, the solution set of $x^2 + 2x - 15 = 0$ is $\{-5, +3\}$.

2) $x^2 + 5x = -6 \quad \Rightarrow \quad (x + 2)(x + 3) = 0$

Step 1: $x^2 + 5x + 6 = 0$ By writing the quadratic equation in standard form.

Step 2: The last term is +6, so we will look for factors of +6 that when we get the sum will give us the middle term +5. The factors of +6 are $(-1)(-6), (1)(6), (-2)(-3), \text{and } (2)(3)$. However, it is the fourth pair that will satisfy the middle term.

Step 3. By the Zero Product Property, $(x + 2)(x + 3) = 0$ implies $(x + 2) = 0 \text{ or } (x + 3) = 0$. That is, $x + 2 = 0$ and $x + 3 = 0$.

Step 4: Further, applying addition property of equality we have $(x + 2) + (-2) = 0 + (-2)$. Hence, $x = -2$. And $(x + 3) + (-3) = 0 + (-3)$. Thus, $x = -3$

Therefore, the solution set of $x^2 + 5x = -6$ is $\{-2, -3\}$.

3) $3x^2 - 8 = 10x \quad \Rightarrow \quad (3x + 2)(x - 4) = 0$

Step 1: $3x^2 - 10x - 8 = 0$, by writing $3x^2 - 8 = 10x$ in standard form.

Step 2: The last term is 8, so we will look for factors of +8 that when we get the sum will give us the middle term -10. The factors of -8 are $(2)\text{and }(-4)$

Step 3. $(3x+\ 2)(x-4)=0$ implies $(3x+\ 2)=0$ or $(x-4)\ =0$. Or look for factors of 3 and -8 that when multiplied and combined will give -10. The factors of -8 are 2 and -4 while the factors of 3 are 3 and 1 that will give the desired result. Thus, (-4)(3) + (2)(1) = -10 which is the middle term.

Step 4: Further, $(3x+\ 2)+(-2)=0+(-2)$. So, $\frac{1}{3}(3x=-2)\frac{1}{3}$.

Hence, $x=-\frac{2}{3}$. On the other factor, $(x-4)+(4)=0+(4)$. Thus, $x=4$.

Therefore, the solution set of $3x^2-8=10x$ is $\left\{-\frac{2}{3},4\right\}$.

4) $4x^2+10x-6=0 \implies (2x-1)\,(2x+6)\ =0$ or $2(2x-1)\,(x+3)\ =0$

Step 1: $4x^2+10x-6=0$

Step 2: The last term is -6, so we will look for factors of -6 that when we get the sum will give us the middle term +10. The factors of -6 are $(-1)\text{and }(6)$ which will lead us to the sum of the middle term which is +10 ofcourse with consideration of the numerical coefficient of the first term which is 4.

Step 3. $(2x-1)(2x+6)=0$ implies $(2x-1)=0$ or $(2x+6)\ =0$. Or look for factors of 4 and -6 that when multiplied and combined will give 10. The factors of 4 are 1(4), (-1)(-4), 2(2), (-2)(-2) while the factors of -6 are 1(-6), (-1)(6), 2(-3), (-2)(3). Choose the factors that when multiplied and combined will result to 10. You may do this by trial and error. Thus, (2)(6) + (2)(-1) = 10 which is the middle term.

Step 4: Further, $2x-1+(1)=0+(1)$. So, $\frac{1}{2}(2x=1)\frac{1}{2}$. Hence, $x=\frac{1}{2}$. On the other factor, $(2x+6)+(-6)=0+(-6)$. Further, $\frac{1}{2}(2x=-6)\frac{1}{2}$. Thus, $x=-3$. Therefore, the solution set of $4x^2+10x-6=0$ is $\left\{\frac{1}{2},-3\right\}$.

5) $-t^2+10t=25$

Step 1: $-t^2+10t-25=0$ can be written as $t^2-10t+25=0$ in standard form by multiplying the whole equation by -1.

Step 2: The last term is 25, so we will look for factors of $+25$ that when we get the sum will give us the middle term -10. The factors of $+25$ are $(-5)\text{and }(-5)$ which will lead us to the sum of the middle term which is -10.

Step 3. $(x-5)(x-5)=0$ implies $(x-5)^2=0$. Applying the definition of exponent which says that, when multiplying the same base, just copy the base and simply add their exponents.

Step 4: Further, after factoring, the extraction of square root of both sides should be done. $\sqrt{(x-5)^2} = \sqrt{0}$. So, $x - 5 = 0$. Hence, applying addition property of equality, we have $x = 5$. Therefore, the required time in order that the arrow will be 25 feet above the ground is 5 seconds.

Activity 3: LET'S DO THE FACTORING!

Directions: Solve each of the following quadratic equations by factoring method. Write "not possible" for items that cannot be factored out.

1) $25x^2 - 9 = 0$
2) $x^2 - 20x + 100 = 0$
3) $x^2 + 5x + 1 = 0$
4) $2x^2 - 3 + 5x = 0$
5) $x^2 + 4x = -4$
6) $6x^2 + x - 1 = 0$
7) $x^2 - 5x - 7 = 7$
8) $3x^2 - 3x - 6 = 0$
9) $-2 + x + x^2 =$
10) $2x^2 - x - 1 = 0$

Reflective Questions:

1) How do you find factoring in Activity 3? Can you describe your understanding on how you did the factoring method?
2) Discuss some of the strategies/techniques in factoring.
3) In what particular scenario you can apply the concept of factoring?

Activity 4: LET'S APPLY!

Direction: Solve each of the following word problem.

1. The length of a rectangle is 3 meters less than twice its width. If the area of the rectangle is 135 m^2, find the perimeter of the rectangle.

2. Aaron is driving to a best friend's house. He drives 15 miles north and then drives 18 miles west.
   a. How many miles does Aaron have to drive to get to his best friend's house?
   b. If there were a direct road (straight line) between Aaron's house and his best friend's house, how far would Aaron have to drive?
   c. Which route is shorter? How much shorter is it?

Reflective Questions:

1. How did you solve the word problem?
2. Can you name some mathematical properties and principles applied in getting the factors of expressions? How did you apply it?
3. Give your own example where the concept of factoring is applied.

EXERCISES

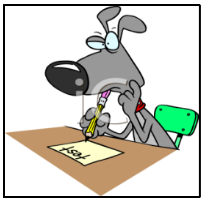
A. Computation. Directions: Determine the roots of each of the following quadratic equations using factoring method.

1) $x^2 - 14x + 49 = 0$
2) $x^2 - 125 = 0$
3) $x^2 + 7x - 30 = 0$
4) $4x^2 - 16 = 0$
5) $x^2 + 5x = -6$
6) $12x^2 + 2x = 0$
7) $5x^2 - 12 = 48$
8) $-3x^2 + 2 = 5x$
9) $x^2 - 2x - 3 = 0$
10) $6x^2 - x - 1 = 0$

B. Word Problem. Solve each of the following.

1. A three-sided polygon has a side with 10 inches and a hypotenuse with length of 18 inches. Find the length of the second leg of this polygon.
2. A ball is shot from a cannon into the air with an upward velocity of 36 ft/sec. The equation that gives the height (h) of the ball at any time (t) is: $h(t) = 6t^2 + 3t - 3$. Find the maximum height attained by the ball.

LET'S WORK THIS OUT



WORKSHEET NO. 3
QUADRATIC EQUATIONS: FACTORING METHOD

Direction: Determine the roots of each of the following quadratic equations using the factoring method. Please write your solution on the space provided.

1. $216x^2 - 27x = 0$	2. $k^2 - k - 72 = 0$

3. $12m^2 - 17m - 5 = 0$	4. $2w^2 - 12w = 2w + 60$

Let's Reflect:

1. How do you find answering this worksheet? (*Please narrate your experience in one sentence.)*

2. Which part of the activity you find it challenging? (*Please specify by writing a sentence the challenges and difficulties encountered*.)

SUMMARY

- Factoring is one of the methods of finding the roots of the quadratic equations.
- It is called factoring since we look for the factors, that is, something we multiply by.
- Some of the strategies in factoring is by looking at common factors between terms of the given expressions, using guess and check, and a method for simple cases.
- Always remember to apply the *Zero Product Principle* when factoring quadratic equation.
- Do not forget to write the quadratic equation in its standard form and equated to zero.

After completing this module, you are task to go online to take the posttest using your *GoConqr* account. Good luck!