Linear Equations in One Variable

A **linear equation in one variable** is an equation involving constants and a single variable which only occurs to the first power. The following equations are linear equations in one variable:

$$2x + 1 = 3$$

$$-4x - 5 = 7x + 13$$

$$\frac{2}{3}x - 7 = \frac{1}{3} - \frac{1}{6}x$$

The problem is to solve for the variable. To do this, you may:

1. Simplify either side alone, using basic arithmetic such as multiplying out or combining terms.

2. Add, subtract, multiply, or divide both sides by the same thing. You may not multiply or divide by 0.

The primary goal is to **try to get all the variable terms on one side and all the number terms on the other**. Once you've done this, the equation can be solved easily.

Example. Solve for x:

$$3x + 7 = 12 - 10x.$$

$$\begin{aligned} 3x + 7 &= 12 - 10x \\ 3x + 7 + 10x - 7 &= 12 - 10x + 10x - 7 \\ 13x &= 5 \\ \frac{13x}{13} &= \frac{5}{13} \\ x &= \frac{5}{13} \quad \square \end{aligned}$$

First, I added $10x$ and subtracted 7 from both sides. Then all the x-terms wound up on one side and all the terms without x wound up on the other side.

Then dividing both sides by 13 solved for x.

While there are many mathematically correct ways to solve an equation, this general approach is often useful: Add or subtract to get the variable terms on one side and the number terms on the other, then divide to solve.

Finally, notice that I left the answer as $\frac{5}{13}$. Unless you are told otherwise, you should leave answers in exact, simplified form: Don't give decimal approximations.

Example. Solve for p:

$$-4p - 5 = 7p + 13.$$

In this example, I'll arrange the arithmetic "vertically". Compare it to the work in the previous example and use whatever approach you like best.

$$\begin{array}{rrrrrrr} & -4p & - & 5 & = & 7p & + & 13 \\ - & & & 5 & & & & 5 \\ \hline & -4p & & & = & 7p & + & 8 \\ - & 7p & & & & 7p & & \\ \hline & -11p & & & = & & & 8 \\ / & -11 & & & & & & -11 \\ \hline & p & & & = & & & -\frac{8}{11} \end{array} \quad \square$$

The variable in this problem was p rather than the more-common x, but it doesn't affect how you solve the equation.

Example. Solve for x:

$$3(4x + 7) = 13 - 23x.$$

In this case, I start by multiplying out $3(4x + 7)$ to eliminate the parentheses. This gives an equation like the ones in the earlier examples.

$$\begin{aligned} 3(4x + 7) &= 13 - 23x \\ 12x + 21 &= 13 - 23x \\ 12x + 21 + 23x - 21 &= 13 - 23x + 23x - 21 \\ 35x &= -8 \\ \frac{35x}{35} &= \frac{-8}{35} \\ x &= -\frac{8}{35} \quad \square \end{aligned}$$

The next two examples have answers that may surprise you.

Example. Solve for x:

$$3(2x + 5) = 17 + 6x.$$

$$\begin{aligned} 3(2x + 5) &= 17 + 6x \\ 6x + 15 &= 17 + 6x \\ 6x + 15 - 6x - 15 &= 17 + 6x - 6x - 15 \\ 0 &= 2 \end{aligned}$$

The x's are gone, and the last equation is false (a **contradiction**), since you know that 0 and 2 aren't equal. When this happens, it means that the original equation has **no solutions**.

You can simply write "no solutions" as your answer.

If you know set notation, you can also write "$\emptyset$", the symbol for the empty set.

Don't answer with vague phrases like "can't be done", or nonstandard abbreviations like "DNE".

Example. Solve for x:

$$-12x + 10 = 2(5 - 6x).$$

$$\begin{aligned} -12x + 10 &= 2(5 - 6x) \\ -12x + 10 &= 10 - 12x) \\ -12x + 10 + 12x - 10 &= 10 - 12x + 12x - 10 \\ 0 &= 0 \end{aligned}$$

The x's are gone, and the last equation is true, since 0 is obviously equal to 0. When this happens, it means that the solution to the original equation is **all real numbers**.

You can write your answer as "all real numbers".

If you want to be fancy or save some writing, you may write your answer as "$\mathbb{R}$" (the symbol for the real numbers --- notice the "double bar" on the "$\mathbb{R}$").

You could also write "$(-\infty, \infty)$" (interval notation).

In some cases, the numbers in the equation are fractions or decimals. While you can solve these equations using the same approach as in the previous examples, it may be easier to eliminate the fractions or decimals first. This makes it less likely that you'll make arithmetic mistakes.

Example. Solve for x:

$$\frac{7}{9}x - \frac{5}{2} = \frac{7}{6}(3x - 10).$$

If an equation has fractions, you can multiply both sides by the least common multiple of the denominators to clear the denominators.

The denominators of the fractions are 9, 2, and 6. To find the least common multiple, take the largest number 9. Take multiples of 9 until you find one that is divisible by 9, 2, and 6.

Now 9 itself is not divisible by 2 or 6. Take the next multiple of 9, which is 18. Since 18 is divisible by 2 and 6, it is the least common multiple. Therefore, I'll multiply both sides of the equation by 18:

$$\begin{aligned} \frac{7}{9}x - \frac{5}{2} &= \frac{7}{6}(3x - 10) \\ 18 \cdot \left(\frac{7}{9}x - \frac{5}{2}\right) &= 18 \cdot \frac{7}{6}(3x - 10) \\ 18 \cdot \frac{7}{9}x - 18 \cdot \frac{5}{2} &= 18 \cdot \frac{7}{6}(3x - 10) \\ 14x - 45 &= 21(3x - 10) \\ 14x - 45 &= 63x - 210 \\ 14x - 45 - 14x + 210 &= 63x - 210 - 14x + 210 \\ 165 &= 49x \\ \frac{165}{49} &= \frac{49x}{49} \\ \frac{165}{49} &= x \quad \square \end{aligned}$$

Example. Solve for x:

$$0.23x + 0.4 = 5(6 - 0.7x).$$

If an equation has decimals, you can multiply by 10, 100, 1000, ... to get rid of the decimals.

The "0.23" has the largest number of digits (2 digits) to the right of the decimal point. So to clear the decimals, I need to multiply both sides by $10^2 = 100$ (think "2 zeroes").

$$\begin{aligned} 0.23x + 0.4 &= 5(6 - 0.7x) \\ 100 \cdot (0.23x + 0.4) &= 100 \cdot 5(6 - 0.7x) \\ 100 \cdot (0.23x + 0.4) &= 5 \cdot 100(6 - 0.7x) \\ 100 \cdot 0.23x + 100 \cdot 0.4 &= 5 \cdot (100 \cdot 6 - 100 \cdot 0.7x) \\ 23x + 40 &= 5(600 - 70x) \\ 23x + 40 &= 5 \cdot 600 - 5 \cdot 70x \\ 23x + 40 &= 3000 - 350x \\ 23x + 40 - 40 + 350x &= 3000 - 350x - 40 + 350x \\ 373x &= 2960 \\ \frac{373x}{373} &= \frac{2960}{373} \\ x &= \frac{2960}{373} \quad \square \end{aligned}$$

Example. Solve for w:

$$3(w + 2) - 1 = 2(2w + 1) - 4.$$

First, I'll multiply out and combine terms:

$$\begin{aligned} 3(w + 2) - 1 &= 2(2w + 1) - 4 \\ 3w + 6 - 1 &= 4w + 2 - 4 \\ 3w + 5 &= 4w - 2 \end{aligned}$$

Next, get all the w's on one side and all the numbers on the other. For variety, I'll arrange the algebra vertically:

$$\begin{array}{rrrrrrr} & 3w & + & 5 & = & 4w & - & 2 \\ + & & & 2 & & & & 2 \\ \hline & 3w & + & 7 & = & 4w & & \\ - & 3w & & & & 3w & & \\ \hline & & & 7 & = & w & & \end{array} \quad \square$$

Contact information

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