Chapter 6.6-7: Work, Energy, Moments and Center of Gravity

6.6 Work

$$W = F \times d$$

SI Units:
$N - - - > M - - - > J$
CGS
$Dyne - - - > cm - - - > erg$
BE:
$lb - - - > ft - - - > ft - lb$

$$W = \int_a^b F(x)dx$$

$F(b-a)$

Hookes Law: Think, of captain hook, sailing on a spring
$F(x) = kx$
$k =$ spring constant
$x =$ distance stretched

F(x) will be in units N, dyne, or lb

Review: 6.1 # 12, 6.2 #4, 6.3 #10

6.1 #12
$x^2 = y$
$x = y - 2$
$x^2 = x + 2$
$x^2 - x - 2 = o$

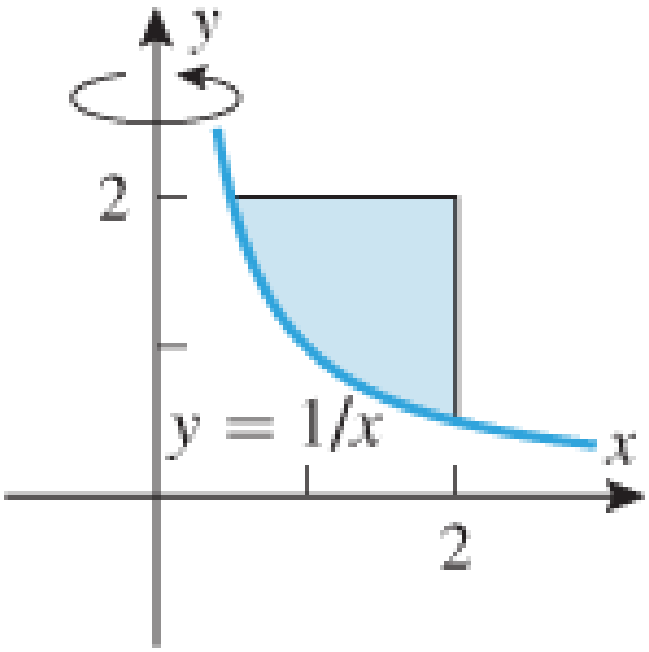
$(x-2)(x+1)$

$x=2\ ;x=\ -1$

$$\int_{-1}^{2}\left[\left[x+2\right]-\left[x^{2}\right]\right]dx$$

6.2 #4

What direction are we taking slicing?
the one that forms disks...
Rotate around $y-$axis



Find Volume

$\pi(f(x^{2})-g^{2}(x))$

$$\int_{\frac{1}{2}}^{2}\pi\left(\left(2^{2}\right)-\left(\frac{1}{y}\right)^{2}\right)dy$$

$$\int_{\frac{1}{2}}^{2}\pi\left(4-\frac{1}{y^{2}}\right)dy=\ \pi\left(4y+\frac{1}{y}\right)\ \text{evaluated at}\ \left[2,\ \frac{1}{2}\right]=\ \frac{9\pi}{2}$$

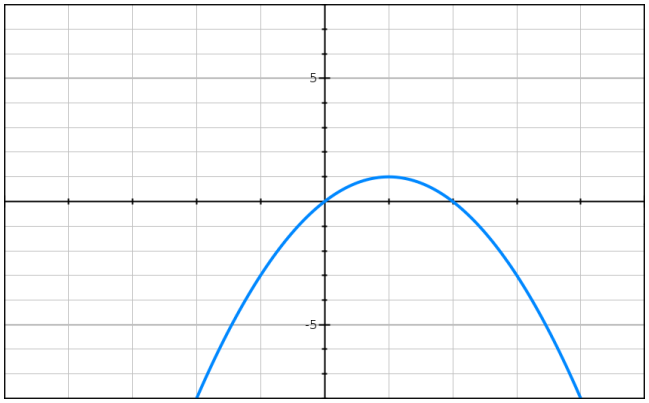
6.3 #10

Find the Volume using cylindrical shells

$y = 2x - x^2 = x(2 - x)$

$y = 0$

DRAW IT OUT



Surface Area:

$2\pi r \times h$

$r = x$

int $2\pi x( )$

To Determine the Force in these Problems: Use Hooks Law

$F(x) = kx$

Scientific Method:

1. Observation
2. Hypothesis
3. experiment
4. Measure results aka, Data
5. Accept, or reject
6. Refine

These together are a scientific theory: collection of verified Hypothesese

Imagine we have a petri dish – Bacteria grows to grow and grow

Let us drop just 1 bacteria, What happens?

Exponential GROWTHHHH!

We use this observation to form a hypothesis

$y(t) = 2^t$ just a simple example

276 years ago Calculus was invented

It was all about rates of change

To find this rate of change, we take the derivative $y'(t)$

$2 =$ rate of change over a point in time but we actually use

$y'(t) = k \times y(t)$ becoming a differential equation

$$\int \frac{1}{y(t)} \times y'(t)dt = \int kdt$$

$$e^{\ln(y(t))} = e^{kt+c}$$

$$y(t) = e^{kt+C}$$

$$= e^c \times e^{k\times}$$

$$y(t) = ae^{kt}$$

$$\lim_{f > inf} ae^{kt} = \infty$$

Ex. Imagine we have a graph with $x - ax =$ $y(t)$ and $y'(t)$

$y'(t) = y(t)(1 - y(t))$

At $x = 1$ we are at $y = 0$ at 1.

$$\int \frac{1}{y(1-y)} y'dt = \int 1dt$$

$$\int\left(\frac{1-y+y}{y(1-y)}\right) = t + c$$

$$= t + C$$

$$\int\left(\frac{1}{y} + \frac{1}{1-y}\right)dt = t + C$$

$$\ln(y) - \ln(1-y) = t + C$$

$$\ln\left(\frac{y}{1-y}\right) = t + C$$

$$= ae^{kt}$$

Calculate the limit:

$$\lim_{t \text{ to } \infty} \frac{ae^t}{1+ae^t} = \lim_{t \text{ to } \infty} \frac{ae^t}{ae^t} = \lim_{t \text{ to } \infty} 1 = 1$$

Common Uses:
Introduce Reactant to a batch,
Making drugs
Linguistists – spread of new words
Economics – Spread of trends in the market
Model should match reality, some times it changes, we can alter our rate of change equation to match these changes.
These will be solved through Series of Something....

6.2 Wiley # 24
Must

h(x)

$$\int \pi\left(\sqrt{x} + 2\right) - \left((x+2)^2\right) dx$$

$$x = 0$$

$$x = 1$$

Shifting a picture:

$g(x+2)$: Left

$g(x-2)$: right

$g(x)+2$: up 2

$g(x)-2$: down 2