$\Rightarrow$ If you want to report error limits on the calculated mean of a measured quantity, be clear about what your error limits mean.

Example: $X = 48.2 \pm 0.6$

This could be $\pm \frac{1}{2}R$, $\pm \sigma_X$, $\pm 2\sigma_X$, or $\pm 3\sigma_X$

Solution $X = 48.2 \pm 0.6 \ (2\sigma_X)$

Dimensional Homogeneity and Dimensionless Quantities

$\Rightarrow$ All additive terms on both sides of a valid equation must have the same dimensions. This is the principle of dimensional homogeneity.

$$u\,(m/s) = u_0\,(m/s) + g\,(m/s^2)\, t\,(s)$$

each additive term has units of length/time.

$$u = u_0 + g$$

each term doesn't have the same units, so the eqn isn't valid.

$\Rightarrow$ If an eqn is dimensionally homogeneous, but the additive terms have inconsistent units, they can be made consistent by applying the appropriate conversion factors.

$$u\,(m/s) = u_0\,(m/s) + g\,(m/s^2)\, t\,(min)\,(60\ s/min)$$

inconsistent time units

appropriate conversion factor

Example 2.6-1

Consider the eqn $D(ft) = 3t(s) + 4$

1. If the eqn is valid, what are the dimensions of the constants 3 & 4?

   $\Rightarrow$ Each term must have units of (ft) for the eqn to be valid.

   $\therefore$ 3 must have units of length / time

   4 must have units of length

2. If the eqn is consistent in its units, what are the units of 3 & 4?

   $\Rightarrow$ 3 must have units of ft/s.

   4 must have units of ft.

3. Derive an eqn for distance in meters in terms of time in minutes.

$$\Rightarrow \quad D(ft) = \frac{D'(m) \,\big|\, 3.2808\ ft}{\big|\, m} = 3.28\,D'(m)$$

$$t(s) = \frac{t'(min) \,\big|\, 60\ s}{\big|\, min} = 60\,t'(min)$$

Substitution gives,

$$3.28\,D'(m) = 3\,(60)\,t'(min) + 4$$

$$\Rightarrow D'(m) = 55\,t'(min) + 1.22$$

General Procedure for Converting Eqn into Variables w/ Different Units

1. Define variable w/ different units
2. Define variable w/ original units as function of variable w/ different units.

3. Substitute these functions into original equation & simplify.

Dimensionless Quantity

⇒ Can be a pure number $(2, \frac{5}{2})$ or a multiplicative combination of variables with no net dimensions:

$$\frac{M\,(g)}{M_0\,(g)} \quad \text{or} \quad \frac{\rho\,(g/cm^3)\;v\,(cm/s)\;D\,(cm)}{\mu\,(g/cm\text{-}s)}$$

Called Dimensionless Groups

⇒ Exponents, Transcendental Functions (e.g. log, exp ≡ e and sin), and arguments of these functions (e.g. x in $\sin x$) must be dimensionless quantities.

Example 2.6-2

Quantity k depends on temperature T as follows:

$$k\left(\frac{\text{gmole}}{cm^3\text{-}s}\right) = 1.2 \times 10^5 \exp\left\{\frac{-20000}{1.987\,T}\right\}$$

⇒ 20,000 has units of cal/gmole.
T has units of °K.

What are the units of 1.2×10^5 & 1.987?

⇒ Quantity in exponential is dimensionless

$$\therefore \quad \frac{20000 \text{ cal}}{\text{gmole}} \bigg| \frac{}{T\ °K} \bigg| \frac{\text{gmole-}°K}{1.987 \text{ cal}}$$

← units required to cancel other units.

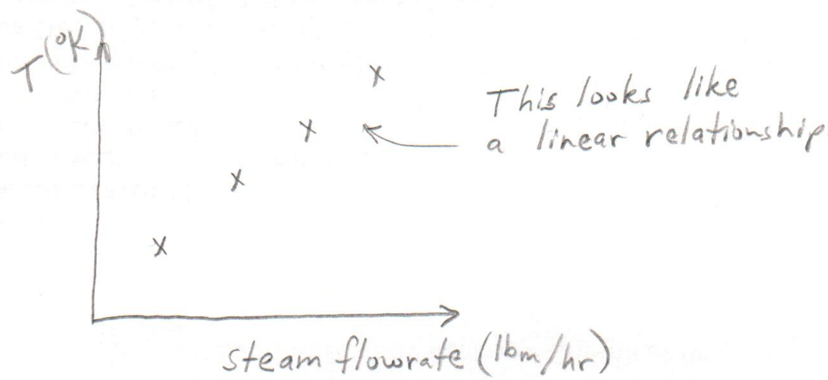
∴ Units of 1.2×10^5 & k are the same

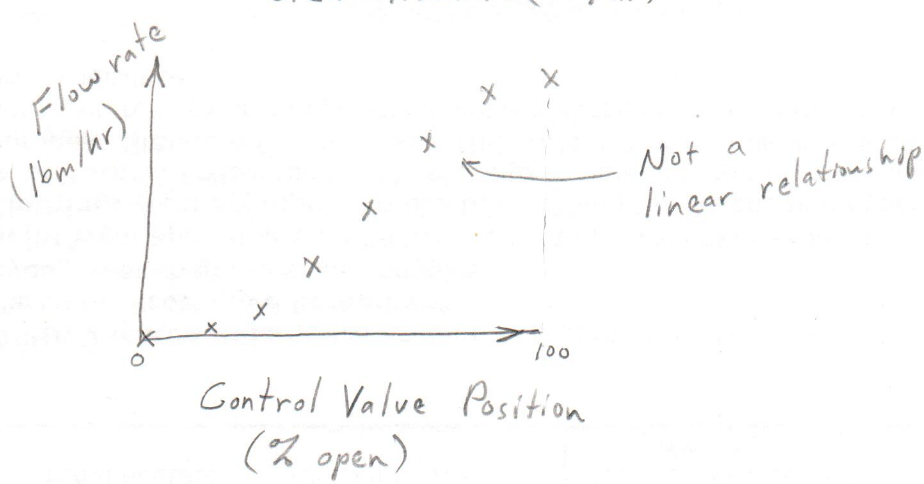
Answer: $1.2 \times 10^5 \ \text{gmole}/\text{cm}^3\text{-s}$ & $1.987 \ \text{cal}/\text{gmole-}^\circ\text{K}$

Process Data Representation & Analysis

Operation of any process is based upon measurements of several process variables - temperature, pressure, flow rate, concentration, etc.

Developing accurate representations between these measured variables and the quantities that affect them will determine how well the process operates.





Interpolation and Curve Fitting Techniques are used to describe these relationships at points other than those measured.

Two-Point Linear Interpolation

⇒ Use to estimate y at points between measured data points.

$$y = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

⇒ Not recommended for extrapolation.

Fitting Data to a Straight Line

If a plot of the measured quantities suggests a linear relationship, then use the linear least-squares relationship to determine the slope and intercept.

$$y = ax + b$$

Linear Least-Squares Relation:

$$a = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

$$b = \frac{1}{n}\left\{\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i\right\}$$

<u>NOTE</u>: This isn't in the book!!! (It's in Appendix A).

Example 2.7-1

Rotameter Calibration Data

Rotameter Reading R	Flowrate $\dot{V}$ (L/min)
10	20
30	52.1
50	84.6
70	118.3
90	151.0

$$\sum_{i=1}^{n} x_i y_i = 27{,}864 \qquad \sum_{i=1}^{n} x_i = 250$$

$$\sum_{i=1}^{n} y_i = 426 \qquad \sum_{i=1}^{n} x_i^2 = 16500$$

$$n = 5$$

$$a = \frac{27{,}864 - (250)(426)/5}{16500 - (250)^2/5} = 1.641$$

$$b = \{426 - (1.641)(250)\}/5 = 3.15$$

Fitting Non-linear Data

⇒ With some non-linear equations you can still use the linear least-squares fitting method.

$$\underbrace{y^2}_{y'} = a \underbrace{x^3}_{x'} + b$$

⇒ In general, you apply the linear least-squares relationship if any two quantities can be related by an equation of the form

$$(\text{Quantity } 1) = a(\text{Quantity } 2) + b$$

Example 2.7-2

A mass flow rate $\mathring{m}$ (g/s) is measured as a function of temperature T (°C).

T	$\mathring{m}$
10	14.76
20	20.14
40	27.73
80	38.47

There is reason to believe $\mathring{m}$ varies linearly w/ $T^{1/2}$:

$$\mathring{m} = aT^{1/2} + b$$

Use least-squares relationship to determine a & b.

$$y \equiv \mathring{m} \qquad x \equiv T^{1/2}$$

$$\sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} T_i^{1/2} \mathring{m}_i = 656.21$$

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} T_i^{1/2} = 22.90$$

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \mathring{m}_i = 101.1$$

$$\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} T_i = 150 \qquad n = 4$$

$$a = \frac{656.21 - (22.90)(101.1)/4}{150 - (22.90)^2/4} = 4.10$$

$$b = \left\{101.1 - (22.90)(4.10)\right\}\left(\frac{1}{4}\right) = 1.80$$

⇒ Two non-linear functions that often occur in process analysis are

Exponential function: $y = a e^{bx}$

Power Law function: $y = a x^{b}$

Q: Can we manipulate these into a form so the linear least-squares relationship can be applied?

A: Yes

Take natural log of both sides:

Exponential function: $$\underset{\overset{\equiv}{y'}}{\ln y} = \underset{\overset{\equiv}{a'}}{\ln a} + bx$$

Power-Law function: $$\underset{\overset{\equiv}{y'}}{\ln y} = \underset{\overset{\equiv}{a'}}{\ln a} + b\underset{\overset{\equiv}{x'}}{\ln x}$$

KEY

Quiz #1 – February 14, 2014
Please show all work and clearly state any assumptions.

Given the following equation describing the vapor pressure (P^{sat}) of water as a function of temperature (°C) from 1°C to 100 °C:

$$P^{sat}(mm\ Hg) = 10^{\left\{8.07131 - \frac{1730.63}{T(°C) + 233.426}\right\}}$$

a) What are the units for the parameters 10, 8.07131, 1730.63, and 233.426 in the equation above?

10 mm Hg, 8.07131 dimensionless, 1730.63 °C, 233.426 °C

b) Convert the equation so that P^{sat} is in atm and T is in °F. Note that 760mm Hg = 1 atm and T(°C) = (T(°F) – 32)/1.8.

Convert $[P^{sat}(atm)]'$ to $P^{sat}(mm\ Hg)$:

$$[P^{sat}(atm)]' \left| \frac{760\ mm\ Hg}{atm} \right. = P^{sat}(mm\ Hg)$$

We already have $T'(°F)$ converted $T(°C)$

Substitute For $P^{sat}(mm\ Hg)$ And $T(°C)$:

$$760[P^{sat}(atm)]' = 10^{\left\{8.07131 - \frac{1730.63}{(T'(°F) - 32)/1.8 + 233.426}\right\}}$$

$$P^{sat}(atm) = \left(\frac{1}{760}\right) 10^{\left\{8.07131 - \frac{1.8(1730.63)}{T(°F) - 32 + (1.8)(233.426)}\right\}}$$

$$= \left(\frac{1}{760}\right) 10^{\left\{8.07131 - \frac{3115.34}{T(°F) + 388.1668}\right\}}$$

You could have stopped here

$$= 10^{\left\{8.07131 + \log_{10}\left(\frac{1}{760}\right) - \frac{3115.34}{T(°F) + 388.1668}\right\}}$$

$$= 10^{\left\{5.19050 - \frac{3115.34}{T(°F) + 388.1668}\right\}}$$