AMPERE'S LAW



Introduction

- A useful law that relates the net magnetic field along a closed loop to the electric current passing through the loop.
- First discovered by André-Marie Ampère in 1826



ARC

Definition

• The integral around a closed path of the component of the magnetic field tangent to the direction of the path equals μ_0 times the current intercepted by the area within the path

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I$$



Or, in a simplified scalar form

$$\oint B_{||} \, ds = \mu_0 I$$



Thus the line integral

(circulation) of the magnetic field around

some arbitrary closed curve is proportional to

the total current enclosed by that curve

ARC.

Important Notes

- In order to apply Ampère's Law all currents have to be steady (i.e. do not change with time)
- Only currents crossing the area *inside* the path are taken into account and have some contribution to the magnetic field
- Currents have to be taken with their algebraic signs (those going "out" of the surface are positive, those going "in" are negative)- use right hand's rule to determine directions and signs

THE

- The total magnetic circulation is zero only in the following cases:
- -the enclosed net current is zero
- -the magnetic field is normal to the selected path at any point
- -the magnetic field is zero
- Ampère's Law can be useful when calculating magnetic fields of current distributions with a high degree of symmetry (similar to symmetrical charge distributions in the case of Gauss' Law)

THE

Example: Calculating Line Integrals

The figure below shows two closed paths wrapped around two conducting loops carrying currents i_1 and i_2 . What is the value of the integral for (a) path 1 and (b) path 2?



To do this you have to use the right hand rule to check whether the currents are positive or negative relative to the path. On path 1 i_1 penetrates in the negative direction while i_2 penetrates in the positive direction, so $\int \mathbf{B} \cdot d\mathbf{s} = \mu_o \left(i_2 - i_1\right)$.

ARC

On path 2 i₁ penetrates twice in the negative direction and i₂ once in the negative direction so $\int \mathbf{B} \cdot d\mathbf{s} = -\mu_o \left(2i_1 + i_2\right)$

Example: Coaxial Cable

A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 distributed uniformly across their cross-sections. Determine the magnitude and direction of the magnetic field at a distance r from the axis. Make a graph of the magnitude of the magnetic field as a function of the distance r from the axis.



(a) r < a; (b) a < r < b; (c) b < r < c; (d) r > c.



Solution

(a) The enclosed current is $I_{enc} = I_0 \left(\frac{\pi r^2}{\pi a^2}\right) = \frac{I_0 r^2}{a^2}$. Applying Ampere's law, we have

 $B(2\pi r) = \mu_0 \frac{I_0 r^2}{a^2}$ or $B = \frac{\mu_0 I_0}{2\pi a^2} r$, running counterclockwise when viewed from left

(b) The enclosed current is $I_{enc} = I_0$. Applying Ampere's law, we obtain

$$B(2\pi r) = \mu_0 I_0$$
 or $B = \frac{\mu_0 I_0}{2\pi r}$, running counterclockwise when viewed from left



(c)
$$I_{enc} = I_0 - I_0 \left(\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2} \right) = \frac{I_0 (c^2 - r^2)}{c^2 - b^2}$$

Applying Ampere's law,



(d)

$$B = 0$$
 since $I_{enc} = 0$



Example: Cylindrical Conductor

Consider an infinitely long, cylindrical conductor of radius *R* carrying a current *I* with a *non-uniform* current density $J = \alpha r^2$, where α is a constant and *r* is the distance from the center of the cylinder.

(a) Find the magnetic field everywhere.

(b) Plot the magnitude of the magnetic field as a function of r.



Solution

(a) The enclosed current is given by

$$I_{enc} = \int \vec{J} \cdot d\vec{A} = \int (\alpha r'^{2})(2\pi r' dr') = \int 2\pi \alpha r'^{3} dr'$$

For r < R,

$$I_{enc} = \int_{0}^{r} 2\pi\alpha r'^{3} dr' = \frac{\pi\alpha r^{4}}{2}$$

Applying Ampere's law, the magnetic field is given by

$$B(2\pi r) = \frac{\mu_0 \pi \alpha r^4}{2}$$

or

$$B = \frac{\mu_0 \alpha}{4} r^3$$

THE

ARC

For r > R,

$$I_{enc} = \int_{0}^{R} 2\pi\alpha r'^{3} dr' = \frac{\pi\alpha R^{4}}{2}$$

Applying Ampere's law, the magnetic field is given by

$$B(2\pi r) = \frac{\mu_0 \pi \alpha R^4}{2}$$

or

$$B = \frac{\mu_0 \alpha R^4}{4r}$$



(b) Plot the magnitude of the magnetic field as a function of r.





Example: Two Long Solenoids

Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current *I* flowing in each turn, *but in opposite directions*, as indicated on the sketch.



ARC

Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions. Be sure to show your Amperian loops and all your calculations.

a)
$$0 < r < R_1$$

b) $R_1 < r < R_2$
c) $R_2 < r$



Solution

(a) $0 < r < R_1$;

To solve for the magnetic field in this case, we take the top rectangular loop shown in the figure. The current through the loop is

 $I_{\rm enc} = -n_1 \ell I + n_2 \ell I = (-n_1 + n_2) \ell I$





The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$, and $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$. On the other hand, along the outer vertical segment, $\vec{\mathbf{B}} = 0$. Thus, using Ampere's law $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$, we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0 = B\ell = \mu_0 \left(-n_1\ell I + n_2\ell I \right) \implies \vec{\mathbf{B}} = \mu_0 I \left(-n_1 + n_2 \right) \hat{\mathbf{k}}$$



(b) $R_1 < r < R_2$

To solve for the magnetic field in this case, we take the bottom rectangular loop shown in the figure. The current through the loop is

 $I_{\rm enc} = n_2 \ell I$

The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$, and $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$. On the other hand, along the outer vertical segment, $\vec{\mathbf{B}} = 0$. Thus, using Ampere's law $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$, we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0 = B\ell = \mu_0 n_2 \ell I \implies \vec{\mathbf{B}} = \mu_0 n_2 I \hat{\mathbf{k}}$$



(c) $R_2 < r$

Since the net current enclosed by the Amperian loop is zero, the magnetic field is zero in this region.



References

• <u>Physics for Engineers and Scientists</u>, Chapter 29 *Hans C. Ohanian, John T. Markert*

• <u>Fundamentals of Physics</u>, Chapter 31 Halliday, Resnick, Walker

<u>http://ocw.mit.edu</u>

